

Total No. of printed pages = 10
3 (Sem 3) PHY M1

2015

PHYSICS

(Major)

Paper : 3.1

(Mathematical Methods-III and Electrostatics)

Full Marks - 60

Time - Three hours

The figures in the margin indicate full marks for the questions.

GROUP-A

(Mathematical Physics)

Marks : 25

1. Answer the following questions : $1 \times 3 = 3$

(a) Define rank of a matrix.

(b) When a symmetric matrix is said to be Hermitian?

(c) Find $\text{adj } A$ if $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(Turn) over

2. Check whether the total angular momentum matrix for an electron given by

$$J^2 = (3\hbar^2/4) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ is}$$

Hermitian and unitary. 2

3. Answer any two of the following questions :

2×5=10

(a) (i) If $A^2 + A - I = O$ find A^{-1} . 1

(ii) Show that the inverse of a matrix is unique. 2

Or

If A, B are two square, symmetric matrices of dimensions $n \times n$ find the condition when the product AB is symmetric.

(iii) Find the rank of the matrix 2

$$\begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{pmatrix}$$

(b) (i) Express the following quadratic form as product of matrices

$$ax^2 + by^2 + 2hxy \quad 2$$

(ii) What is an idempotent matrix ? 1

(iii) Find the value of b if the matrix

$$\begin{pmatrix} 0 & 1 & b \\ -1 & 0 & 4a \\ 2a & 2b & 0 \end{pmatrix}$$

is skew-symmetric. 2

(c) (i) Show how centrifugal force and coriolis force appear in the description of motion of a particle in the frame fixed with respect to rotating earth. 4

(ii) "Sum of finite rotations performed on a rigid body depends on the order of rotations." 1

What operation on two transformation matrices A, B are linked with the statement ? 1

4. Answer either [(a) and (b)] or [(c) and (d)].

(a) (i) Given the matrices

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 3 & -1 & 1 \\ 4 & 2 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

and the equation $AX = C$.

Solve for x, y, z by matrix method. 3

(ii) Test whether the matrix

$$\begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}$$

is orthogonal or not.

2

(b) (i) Consider the following transformation in three dimensions

1+2=3

$$x' = x \cos \theta + y \sin \theta$$

$$y' = -x \sin \theta + y \cos \theta$$

$$z' = z$$

Write down the transformation matrix A.

Show that $A(\theta_1 + \theta_2) = A(\theta_1)A(\theta_2)$.

(ii) Find the inverse of the matrix

2

$$\begin{pmatrix} 2 & -3 \\ 4 & 6 \end{pmatrix}$$

(c) (i) Test whether the following matrices are diagonalizable:

1½+1½=3

$$A = \begin{pmatrix} 1 & 1+i \\ 1-i & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

(ii) State and explain Cayley - Hamilton theorem.

2

(d) (i) Calculate the eigen values of M , M^{-1}

where $M = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ 2+1=3

(ii) If $A = \begin{pmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{pmatrix}$ and I is a unit matrix, show that 2

$$I + A = (I - A) \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

GROUP-B

(Electrostatics)

Marks : 35

5. Choose the correct option : 1×4=4

(a) The induced surface charge q' is related to charge q as (where K = dielectric constant)

(i) $q' = q/k$

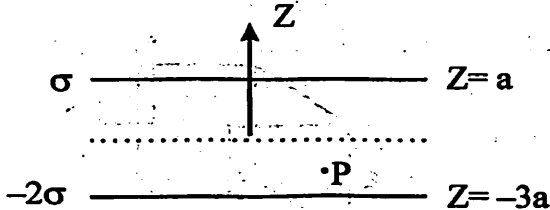
(ii) $q' = q$

(iii) $q' = q \left(1 - \frac{1}{K}\right)$

(iv) $q' = q(1-K)$

Or

What is the net electric field at the point P due to the two infinitely long charged sheets as shown in the figure.



(b) (i) Can the potential have a maximum or minimum in free space? Justify. $1\frac{1}{2}$

(ii) Show mathematically that $\nabla\phi$ is perpendicular to the equipotential surface $\phi(x, y, z) = \text{constant}$. $1\frac{1}{2}$

7. Answer either (a) or (b)

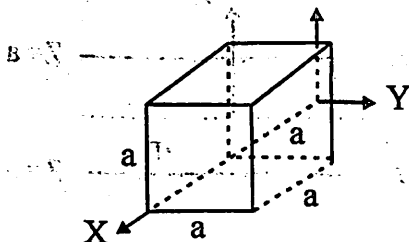
(a) (i) Compute the curl and divergence of electric field given by

$$\vec{E}(x, y, z) = (x^2 + z^2 + 5)\hat{x} + (y^2 + x^2 - 9z)\hat{y} + (z^2 + y^2)\hat{z}$$

Is the field conservative? $1+1+1=3$

(ii) For the electric field $\vec{E} = (Kx^{1/2})\hat{x}$

where K is a constant, compute the electric flux through the face of the cube region depicted in the figure. 2



- (b) (i) Using Gauss's law find an expression for the mechanical force per unit area on the surface of a charged conductor. 2½
- (ii) Derive an expression for the equilibrium of an electrified soap bubble. 2½

8. Answer any two questions :

- (a) (i) Four charges $q_1 = q_2 = -q_3 = -q_4 = q$ are arranged at points $(0, a)$, $(0, -a)$, $(a, 0)$ and $(-a, 0)$ in x - y plane. What is the electrostatic energy needed to compose such an arrangement, bringing the charges in from infinity? Also find the electrostatic potential in x - y plane.

2+3=5

(ii) Show that an arbitrary distribution of charges is equivalent to combination of monopole, dipole, quadrupole etc. 5

(b) (i) Two concentric spherical shells of radii a and b ($b > a$) are charged to potentials V_1 and V_2 respectively. Use Laplace's equation to calculate the electric potential in the region between the shells. 5

(ii) Use Poisson's equation to find the charge density in a region where the potential function is

$$V = a + 4b(x^2 + y^2) - c \log(x^2 + y^2)^{3/2}.$$

5

(c) (i) What is meant by dielectric polarisation? Show how \vec{E} , \vec{D} , \vec{P} are related for an isotropic dielectric medium. Is water molecule a polar molecule? If so, why?

$$1+3+2=6$$

(ii) Using Clausius-Mosotti relation show how can the atomic radius be determined from dielectric constant. 4

(d)(i) Determine the electric potential at a point on the axis of a charged disc of radius a and surface charge density σ . Show that the disc may be regarded as a point charge for far off points. $3+2=5$

(ii) Two equal point charges q are placed at equal distance b from the centre of a grounded conducting sphere of radius a where $b < a$. Calculate the force acting on each of the charges. 5