2014

PHYSICS

(Major)

Paper : 5.2

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option : 1×7=7

(a) The maximum number of electrons in the *d*-subshell of an atom is

(i) 2

(ii) 8

(iii) 10

(iv) 18

(b) Into how many components the $2S_{1/2}$ level of Na may split when a weak magnetic field is applied to result in anomalous Zeeman effect?

(i) 2

(ii) 3

(iii) 4

(iv) 5

(c) From which of the following concepts an explanation of the Bohr quantum condition $L_n = N\frac{h}{2\pi}$ may be found?

(i) Rayleigh scattering

(ii) Planck's idea of quantum

(iii) Louis de Broglie matter wave

(iv) Raman effect

(d) An atom emits X-rays when an orbital electron makes a transition from the kth to the nth orbit, where $k > n$. Which of the following is the most probable case of X-ray emission?

(i) $n = 1,\ k = 2$

(ii) $n = 1,\ k = \infty$

(iii) $n = 4,\ k = 8$

(iv) $n = 4,\ k = \infty$

(e) A high-energy electron strikes a metal of high atomic number. If Q_L, Q_X, Q_S and Q_H be the amounts of energy that appear as light, X-rays, sound and heat respectively, then which of the following is correct?

(i) $Q_L = Q_S = 0$

(ii) $Q_S = 0,\ Q_H < Q_X$

(iii) $Q_L \neq 0,\ Q_H > Q_X$

(iv) $Q_X > Q_L,\ Q_H = 0$

(f) The energy levels of an atom are –20 eV, –10 eV, –5 eV, –2 eV and –0·5 eV, ... A high-energy electron accelerated through a potential difference V strikes and ionizes the atom. The value of V is

(i) 20 volt

(ii) 19·5 volt

(iii) 10 volt

(iv) 12 volt

(g) Which of the following cannot emit visible radiations?

(i) H

(ii) He

(iii) He^{+}

(iv) He^{++}

2. Answer any *four* of the following questions : 2×4=8

(a) In Rutherford α-scattering experiment, it was found that 1×10^{6} particles were scattered at an angle 60° per minute. How many α-particles per minute were observed at an angle 180°?

(b) An electron in motion is equivalent to an electric current. Show that the electronic current I in a hydrogen-like atom is $I \propto \frac{Z^2}{n^3}$, where the symbols have their usual meanings.

(c) Mention two differences as regards wavelength and intensity of observed light in case of Rayleigh scattering and Raman effect.

(d) An X-ray tube operates at 40 kV and the partial vacuum inside the tube offers a resistance of 5 MΩ. How many electrons strike the target per second?

(e) The wavelength of the spectral lines emitted by a hydrogen-like atom for a transition $a \rightarrow b$ is given by

$$\lambda = \frac{120a^2}{a^2 - b^2} \text{ nm}$$

where $a > b$ and $b = 1$. What are the shortest and the longest wavelengths emitted by the atom?

(f) A particle of mass $3 \cdot 2 \times 10^{-27}$ kg passes through the velocity selector of a Bainbridge mass spectrograph. The electric field and the magnetic field used

in the velocity selector are respectively 30 kV m^{-1} and 0·1T. What is the kinetic energy, in eV, with which the particle enters the evacuated D-shaped chamber of the spectrograph?

3. Answer *(a)* and any *two* from *(b)*, *(c)* and *(d)* : 5×3=15

(a) Describe Paschen-Back effect, and show that using proper selection rules, one can get spectral lines corresponding to normal Zeeman effect.

(b) Show that the radii of stable orbits in a hydrogen-like atom are proportional to n^2 / Z, where n is the principal quantum number and Z is the atomic number.

(c) In an experiment on normal Zeeman effect, light of wavelength 600 nm was used. The applied magnetic field was $\frac{\pi}{10} T$. It was seen that the wavelength separation between the two component lines was 0·0102 nm. What was the value of e/m of electron determined from the experiment?

(d) Write a short note on any *one* of the following :

(i) Alkali spectra

(ii) Sommerfeld's relativistic correction of Bohr's atom model

(iii) X-ray spectra

4. Answer *(a)* and *(b)*, and any *one* from *(c)*, *(d)* and *(e)* : 10×3=30

(a) Draw a neat diagram of the experimental arrangement of Stern and Gerlach. What effect the magnetic field would have produced had it been uniform? Justify your answer mathematically. Show how two traces are produced by the atomic beam. 2+1+2+5=10

(b) A photon of X-ray with wavelength λ is incident on a free electron at rest. After the collision with the electron the photon is scattered at an angle α and its wavelength becomes λ', where $\lambda' > \lambda$. Show that $(\lambda' - \lambda)$ is proportional to $\sin^2\left(\frac{\alpha}{2}\right)$, assuming conservation of energy and the electron to recoil with relativistic speed. Is it possible for the electron to absorb the entire energy of the photon? Justify your answer mathematically. 7+1+2=10

(c) Describe Aston's mass spectrograph and discuss how the beam of positive ions is focussed. 2+8=10

(d) What do you mean by impact parameter? What happens when impact parameter becomes zero? A slow-moving proton is shot into a heavy atom at rest. The impact parameter of the collision is b. If the proton is scattered at an angle θ, then show that $\cot\frac{\theta}{2} \propto b$. Is it possible for the target atom to emit X-rays after the impact? Justify your answer. 1+1+6+1+1=10

(e) Write short notes on any *two* of the following : 5×2=10

(i) Raman effect

(ii) Moseley's law

(iii) *L-S* and *J-J* coupling

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2014

PHYSICS

(Major)

Paper : 5.3

(Quantum Mechanics and Astrophysics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Write the answers to the two Groups in separate books

GROUP—A

(Quantum Mechanics)

(*Marks* : 40)

1. Choose the correct answer from the given alternatives (any *four*) : 1×4=4

(a) The failure of the classical wave theory to account the distribution of energy in the spectrum of a blackbody radiation was due to the assumption that, radiation energy is

(i) continuous

(ii) discrete

(iii) mixture of continuous and discrete

(iv) electromagnetic

(b) Davisson and Germer experiment suggests that the electron is

(i) a particle

(ii) a wave

(iii) partly wave and partly particle

(iv) None of the above.

(c) Matter wave functions are

(i) periodic functions

(ii) real functions of x and t

(iii) imaginary functions of x and t

(iv) complex functions of x and t

(d) The value of

$$\left[\hat{x}, \frac{\partial}{\partial x}\right]$$

is

(i) 1

(ii) -1

(iii) $i\hbar$

(iv) $-i$

(e) The wave function $\psi(x)$ is well-behaved if $\psi(x)$ is single-valued and

(i) $|\psi(x)| \to 0$ as $x \to \pm\infty$

(ii) $|\psi(x)| \to 0$ as $x \to 0$

(iii) $|\psi(x)| \to \infty$ as $x \to \pm\infty$

(iv) $|\psi(x)| \to$ finite value as $x \to \pm\infty$

2. Answer any *three* questions : 2×3=6

(a) Explain why we do not observe quantum effects in case of a fast moving cricket ball.

(b) What is the physical significance of the wave function $\psi(x, t)$?

(c) Write down the time-dependent Schrödinger equation for a particle of mass m moving in a potential $v(\vec{r}, t)$.

(d) If ψ is an eigenfunction of both $\hat{\alpha}$ and $\hat{\beta}$, then prove that $[\hat{\alpha}\hat{\beta} - \hat{\beta}\hat{\alpha}] = 0$.

(e) Distinguish between a classical and a quantum harmonic oscillator.

3. Answer any *two* questions : 5×2=10

(a) An incident X-ray photon of frequency ν_0 is scattered by a free electron at rest through an angle ϕ. Using relativistic expression of electron energy, show that the change in the wavelength of the photon is given by

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

where m_0 = rest mass of the electron, h is Planck's constant and c is velocity of light in vacuum. 5

(b) State and explain the de Broglie's hypothesis.

Calculate the de Broglie wavelength of electrons of energy 10^4 eV and compare it with the wavelength of electro-magnetic radiation for which the photon has the same energy. 2+3=5

(c) Using uncertainty relation, show that an electron cannot reside inside a nucleus. A nucleon is confined to a nucleus of diameter 5×10^{-14} m. Calculate the minimum uncertainty in the momentum of the electron and the minimum kinetic energy of the electron. Given, $m_e = 9\cdot1\times10^{-31}$ kg. 2+2+1=5

(d) What do you mean by expectation value of an operator in quantum mechanics? If $\psi(x) = Ae^{-m\omega x^2/\hbar}$, find the expectation values of momentum. 2+3=5

4. Answer *either (a)* and *(b) or (c)* and *(d)* : 5×2=10

(a) A particle on the x-axis has the wave function $\psi(x) = cx^2$ between $x = 0$ and $x = 2$. Normalize the wave function over the interval. Find the probability that the particle can be found between $x = 0\cdot5$ and $x = 0\cdot6$. 2+3=5

(b) State and prove Ehrenfest's theorem. 5

Or

(c) Describe briefly the experiment of G. P. Thomson on the diffraction of electrons.

In the experiment, electrons accelerated by a potential difference of 20 kilo volts are diffracted by a thin metal foil. Calculate the Bragg angle for its first-order diffraction from a set of crystal planes which are 2·0 Å apart. 3+2=5

(d) Show that if $\psi_1(\vec{r})$ and $\psi_2(\vec{r})$ are two independent solutions of the Schrödinger's equation, then

$$\psi(\vec{r}) = a_1\psi_1(\vec{r}) + a_2\psi_2(\vec{r})$$

is also a solution of the Schrödinger's equation. What does it imply? 4+1=5

5. Answer *either (a)* and *(b)* *or (c)* and *(d)* : 5×2=10

(a) Write down the Schrödinger equation for a linear harmonic oscillator. What are the eigenvalues and the eigen-functions of the Hamiltonian of a linear harmonic oscillator? Explain the significance of zero-point energy of the oscillator. 1+2+2=5

(b) State and explain the complementary principle of Niels Bohr. What conclusion can be drawn from the result of γ-ray microscope experiment? 3+2=5

Or

(c) Derive the continuity equation from the time-dependent Schrödinger equation of a particle moving in a real potential. Give the physical interpretation of the continuity equation you derive. 4+1=5

(d) What are conjugate variables in quantum mechanics? Give an example of any one pair of conjugate variables and obtain their commutation relation. 1+1+3=5

GROUP—B

(Astrophysics)

(*Marks* : 20)

6. Answer any *three* of the following : 2×3=6

(a) Define the right ascension and declination of a celestial object.

(b) What do you mean by sidereal time? How is it different from the solar time?

(c) The apparent and absolute magnitudes of a star are +8·6 and +11·4 respectively. Find its distance in parsec.

(d) Find the meridian Zenith distance of Vega ($\delta = +38°44'$) at New Delhi ($\phi = 28°22'$N). Neglect the effect of atmospheric refraction.

7. Answer any *two* of the following : 4×2=8

(a) What is Hertzsprung-Russell diagram? Draw a neat sketch of the H-R diagram showing the position of the main sequence stars with the Sun and the white dwarfs.

(b) Write down the various equations of PP–I, PP–II and PP–III chain reactions that convert four H nuclei into a He nucleus.

(c) Derive a relation between the apparent and absolute magnitudes of a star.

8. Write a short note on any *one* of the following : 6

(a) Pulsars

(b) Black holes

(c) Evolution of the Universe

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[The following data can be used when required :

$[e = 1 \cdot 6 \times 10^{-19}$ C, $c = 3 \times 10^{8}$ m/s

$h = 6 \cdot 6 \times 10^{-34}$ J s, $m_e = 9 \cdot 1 \times 10^{-31}$ kg

$m_p = 1 \cdot 67 \times 10^{-27}$ kg]