2013

PHYSICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Answer the following : 1×3=3

(a) Define self-adjoint matrix.

(b) Define trace of a matrix.

(c) If A is a Hermitian matrix, show that e^{iA} is unitary.

2. Verify that $(AB)^T = B^T A^T$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$$ 2

3. Answer any *two* questions out of *(a)*, *(b)* and *(c)* :

(a) *(i)* If A and B are Hermitian matrices, show that $i(AB - BA)$ is also Hermitian. 1

(ii) If

$$A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

then show that

$$A(\theta)\, A(\phi) = A(\phi)\, A(\theta) = A(\theta + \phi)$$ 2

(iii) Find the value of λ for which the matrix

$$A = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

will be orthogonal. 2

(b) *(i)* Show that inverse of the transpose of a square matrix is the transpose of the inverse. 2

(ii) Prove that any two eigenvectors corresponding to two distinct eigen-values of a unitary matrix are orthogonal. 3

(c) Show that for rotation of one frame with respect to another frame with uniform angular velocity $\vec{\omega}$, the equation of motion of the particle in rotating coordinate system is given by

$$m\frac{d'^2\vec{r}}{dt^2} = \vec{F} - 2m(\vec{\omega} \times \frac{d'\vec{r}}{dt}) - m[\vec{\omega} \times (\vec{\omega} \times \vec{r})]$$

Which term represents the Coriolis force term? Write the equation of motion in case of relative rotation between frames with non-uniform angular velocity $\vec{\omega}$.

3+1+1=5

4. Answer *either* (a) and (b) *or* (c) and (d) :

Either

(a) (i) Prove that the following matrix is unitary : 2

$$\begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$$

(ii) Find the inverse of the following matrix from the adjoint of it : 3

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) (i) Show that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix. 2

(ii) Solve by matrix method the following system of equations : 3

$$x + y + z = 8$$
$$x - y + 2z = 6$$
$$3x + 5y - 7z = 14$$

Or

(c) (i) Show that the eigenvalues of diagonal matrix are precisely the elements in the diagonal. 2

(ii) Given

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

Compute A^{-1} by using the fact that A satisfies its characteristic equation. 3

(d) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{pmatrix}$$ 5

GROUP—B

(Electrostatics)

(*Marks* : 35)

5. Choose the correct option/Answer the following : 1×4=4

(a) When a test charge is brought from infinity along the perpendicular bisector of the dipole, the work done is

(i) positive

(ii) zero

(iii) negative

(iv) None of the above

(b) For a dipole, electric field varies as

(i) r^{-2}

(ii) r^{-3}

(iii) r^{-1}

(iv) r^{-4}

(c) The unit of $\vec{D}$ is

(i) V/m^2

(ii) $coul/m^2$

(iii) V/m

(iv) $coul/m$

(d) What is atomic polarisability?

6. Answer the following questions : $2\times3=6$

(a) Can an electrostatic field have the form $\vec{E} = a(y\vec{a}_x - x\vec{a}_y)$, where a is a constant?

(b) Show that the function

$$\phi = 3x^2 + 8y - 3z^2$$

can represent the electrostatic potential in a charge-free region.

(c) Define relative permittivity. Write down Clausius-Mosotti relation.

7. Find an expression for the electric field at a point on the axis of a uniformly charged disc of radius a and surface charge density σ. Show that the electric field strength at point P inside a spherically symmetric charge distribution is given by

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$$

where R is the radius of the charge distribution and r is the distance of the internal point P from the centre of the charge distribution. $2+3=5$

Or

Find an expression for the torque exerted by one dipole on another dipole. Write down the interaction potential energy of two short electric dipoles separated by a distance. If one of the dipoles is inclined at an angle θ_1 to the radius vector joining them, show that in the state of equilibrium, the other dipole would make an angle θ_2 with it given by

$$\tan\theta_2 = -\frac{1}{2}\tan\theta_1$$

2+3=5

8. Answer any *two* questions out of *(a)*, *(b)*, *(c)* and *(d)* :

(a) *(i)* By using the concept of electrical multipoles, find an expression for the electrostatic potential due to a volume distribution of charge. 5

(ii) An electron can be assumed to be uniformly charged sphere having a total charge e and radius R_0. Show that the electrostatic energy of the electron is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{3}{5}\frac{e^2}{R_0}\right)$$

If this energy is equal to rest energy m_0C^2 of the electron, what must be its radius? 5