2012

PHYSICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Choose the correct option/Answer the following : 1×3=3

(a) What is the modulus of the determinant of a unitary matrix?

(i) 1

(ii) 0

(iii) –1

(iv) None of these

(b) What is a Hermitian matrix?

(c) What is a skew-symmetric matrix?

2. Define conjugate transpose of a matrix. Show that

$$(AB)^+ = B^+ A^+$$ 1+1=2

3. Answer any *two* questions out of *(a)*, *(b)* and *(c)* :

(a) *(i)* For three Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

prove that $\sigma_i \sigma_j = i\sigma_k$, where i, j, k are cyclic permutations of indices. 3

(ii) Show that modulus of each eigenvalue of a unitary matrix is unity. 2

(b) *(i)* Verify that

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

is an orthogonal matrix. 2

(ii) Show that

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}^{-1}$$ 3

(c) What is a frame of reference? A reference frame a rotates with respect to another frame b with uniform angular velocity $\vec{\omega}$. If the position, velocity and acceleration of a particle in frame a are represented by $\vec{r}$, $\vec{V}_a$ and $\vec{f}_a$ respectively, show that the acceleration of that particle in frame b is given by $\vec{f}_b$, where

$$\vec{f}_b = \vec{f}_a + 2\vec{\omega} \times \vec{V}_a + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$ 5

4. Answer *either (a)* and *(b)* or *(c)* and *(d)* :

Either

(a) State Cayley-Hamilton theorem. Obtain the characteristic equation of a matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

and verify Cayley-Hamilton theorem. 5

(b) Find the mutually perpendicular eigenvectors of the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$ 5

Or

(c) Show that the trace of a product of two matrices is independent of the order of multiplication. Also show that eigenvalues of a Hermitian matrix are all real and its eigenvectors corresponding to two distinct eigenvalues are orthogonal. 2+3=5

(d) For the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

determine a matrix P such that $P^{-1}AP$ is a diagonal matrix. 5

GROUP—B

(Electrostatics)

(*Marks* : 35)

5. Choose the correct option/Answer the following : 1×4=4

(a) Electric field vector $\vec{E}$ is

(i) rotational

(ii) irrotational

(b) What do you understand by electrical octupole?

(c) What is meant by electrical image?

(d) Define electrical susceptibility.

6. Answer the following questions : 2×3=6

(a) The electric field due to a short dipole at a point distant 1 cm from it on its perpendicular bisector is $1{\cdot}5\times10^{-11}$ volt/m. Find the dipole moment.

(b) Write down Poisson's equation.

(c) What is a polar molecule? Define molecular polarizability.

7. Write down the integral as well as differential form of Gauss' law. Use this law to show that the expression for field strength at a distance r due to an infinite line charge is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

where λ is linear charge density and r is the distance of the external point from the line charge. 1+1+3=5

Or

Show that the interaction energy of two dipoles of moments $\vec{p}_1$ and $\vec{p}_2$ is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}_1 \cdot \vec{p}_2}{r^3} - \frac{3}{r^5}(\vec{p}_1 \cdot \vec{r})(\vec{p}_2 \cdot \vec{r})\right]$$

where $\vec{r}$ is the radius vector joining the centres of the two dipoles. Hence derive the torque acting on any dipole due to the field of another dipole. 4+1=5

8. Answer any *two* questions out of *(a), (b), (c)* and *(d)* :

(a) *(i)* Show that the electric field due to an electric dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$$

where θ is the angle between $\vec{r}$ and $\vec{p}$. 5

(ii) Show that the energy density of electrostatic field in free space is given by

$$U = \frac{1}{2}\varepsilon_0 E^2$$

where the symbols have got their usual meanings. 5

(b) (i) State and prove the uniqueness theorem regarding solutions to Laplace's equation. 1+4=5

(ii) Use Laplace's equation to find potential inside spherical capacitor. 5

(c) A point charge is situated near an infinite plane earthed conductor. Apply the method of electrical image to calculate—

(i) surface charge density induced on the plane;

(ii) the force between the plane and the charge.

An electron is at a distance 10 Å from an infinite plane conductor. Calculate the force experienced by the electron and the work done in moving it to infinite distance away from the conductor. 4+3+3=10

(d) (i) A spherical cavity is cut in a dielectric medium. Show that

$$\vec{E}_{\text{eff}} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}$$

where the symbols have got their usual meanings. 3

(ii) Deduce Clausius-Mosotti relation. Point out its limitation. 6+1=7

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2012

PHYSICS

(Major)

Paper : 3.2

(Current Electricity and Magnetostatics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) What is the dimension of capacitance in terms of the fundamental quantities (M, L, T, I)?

(b) Write down the differential form of Faraday's law of electromagnetic induction.

(c) What is the SI unit of thermoelectric power?

(d) Show that when a voltage of sinusoidal waveform is applied across a capacitor, the current passing through it leads the voltage by 90°.