2012

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 35)

1. Answer the following questions : 1×4=4

(a) What are coordinate curves in curvilinear coordinate system?

(b) What is the value of $\oint_C \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) d\vec{l}$?

(c) Define scale factors in curvilinear coordinates.

(d) Write the value of $\Gamma(1)$.

2. Answer the following questions : 2×3=6

(a) Evaluate $\int \vec{A} \times \frac{d^2\vec{A}}{dt^2}\, dt$.

(b) Prove that $\oint_C (Mdx + Ndy) = 0$ around every closed curve C in a simply connected region iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ everywhere in the region.

(c) For an orthogonal curvilinear coordinate system, show that $\hat{e}_1 = h_2 h_3 \vec{\nabla} u_2 \times \vec{\nabla} u_3$, where the symbols stand for usual meanings.

3. Find the square of the element of arc length in cylindrical coordinates and hence find the corresponding scale factors. 3+2=5

Or

A fluid of density $\rho(x, y, z, t)$ moves with velocity $\vec{v}(x, y, z, t)$. If there are no sources and sinks, then use divergence theorem to show $\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$, where $\vec{J} = \rho \vec{v}$. 5

4. Answer *either (a) or (b)* and *(c)* :

Either

(a) State and prove Stokes' theorem. 2+8=10

Or

(b) If

$$\vec{A} = (2x^2 + 3y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$$

evaluate $\int_C \vec{A} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $C: x = t,\ y = t^2,\ z = t^3$. 5

(c) If $\vec{F} = 2xz\hat{i} - x\hat{j} + y^2\hat{k}$, evaluate $\iiint_V \vec{F}\, dV$, where V is the region bounded by the surface $x = 0,\ y = 0,\ y = 6,\ z = x^2,\ z = 4$. 5

5. Answer *either (a)* and *(b)* or *(c), (d)* and *(e)* :

Either

(a) Express $\vec{\nabla} \times \vec{A}$ in orthogonal curvilinear coordinates. 7

(b) Show that $\int \delta(a - x)\, \delta(x - b)\, dx = \delta(a - b)$. 3

Or

(c) Express $\vec{\nabla}\phi$ in cylindrical coordinates. 4

(d) Evaluate : 1

$$\int_{-\infty}^{+\infty} (ax^2 + bx + c) \log \frac{x}{2}\, \delta(x - 2)\, dx$$

(e) From the definition of gamma function, establish that $\Gamma(n + 1) = n\Gamma(n)$. Hence find $\Gamma(\frac{7}{2})$ provided $\Gamma(\frac{1}{2}) = \frac{\pi}{2}$. 3+2=5

GROUP—B

(Properties of Matter)

(*Marks* : 25)

6. Answer the following questions : 1×3=3

(a) What is the range of values of Poisson's ratio σ in elasticity?

(b) What is the angle of contact for pure water and clean glass?

(c) What is the dimension of coefficient of viscosity?

7. The pressure inside a soap bubble of radius 1 cm can balance a 1·4 mm column of oil of specific gravity 0·80. Calculate the surface tension of the soap solution ($g = 10\ m/s^2$). 2

8. Answer any *two* of the following questions : 5×2=10

(a) Show that a shear θ is equivalent to half a tensile strain θ/2 and half a compressive strain θ/2 at right angles to each other.

(b) Find the expression for the relation between surface tension and surface energy as $E = S - T\frac{dS}{dT}$, where E, S, T are surface energy, surface tension and absolute temperature respectively.

(c) Consider a fluid having coefficient of viscosity η and density ρ flowing through a narrow cylindrical tube of radius r and length l. If P be the pressure difference in the liquid at the two ends, show that the volume of the fluid flowing in time t is

$$V = \frac{\pi P r^4}{8\eta l} t$$

9. Answer *either (a)* and *(b) or (c)* and *(d)* of the following questions :

Either

(a) Consider a cylindrical rod of length l and radius r, of material of coefficient of rigidity n. If the cylinder is clamped at one end and twisted at the other through an angle θ, then find an expression for twisting couple per unit twist of the cylinder. 7

(b) Show that the isothermal bulk modulus of a gas is equal to its pressure. 3

Or

(c) Find an expression for depression of the free end of a cantilever which is loaded with W and also consider its own weight W_0 as effective. Hence find the expression if it bends under own weight only. 6+1=7

(d) Calculate the amount of work done in stretching by 1 mm a piece of steel wire 1 mm^2 in cross-section and 1 m in length. Young's modulus of steel is 2×10^{11} N/m^2. 3

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