2012

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 20)

1. $\vec{A}$, $\vec{B}$, $\vec{C}$, ... etc., give an algebra. What does it mean? 1

2. (a) How can an elemental area be made a vector quantity? Give an idea that a vector quantity can be associated with electric current. 2

(b) How can a vector field be obtained from a scalar field $\phi(r)$? Can the frictional force be obtained from some potential that way? Give reasons. 1

3\. (a) A function $\phi(r)$ that is even (or odd) under one of these space reflection operations ($x \leftrightarrow -x$ etc.), will remain even (or odd) after the ∇^2 operation but not after the $\vec{\nabla}$ operation. Explain. 2

(b) $\phi(r)$ is a scalar field. State whether the end result in the following cases is scalar or vector : 1

(i) $\nabla^2 \phi(r)$

(ii) $\nabla^2 [\vec{\nabla} \phi(r)]$

(c) Show that

$$\hat{i} \times (\vec{\nabla} \times \vec{r}) \neq (\hat{i} \times \vec{\nabla}) \times \vec{r}$$ 3

Or

If $\phi(r)$ and $\psi(r)$ are two scalar fields such that $\vec{\nabla} \phi(r) \times \vec{\nabla} \psi(r) = 0$ over all spaces, how are their equipotential surfaces and lines of force related?

4\. (a) Give an idea of space curves. How is it useful in the study of kinematics? 2

(b) Show that

$$\nabla^2 f(r) = \frac{d^2}{dr^2} f(r) + \frac{2}{r} \frac{d}{dr} f(r)$$ 5

(c) If $\vec{A}(r)$ is irrotational, show that $\vec{A}(r) \times \vec{r}$ is solenoid. 3

OR

5. (a) $\frac{d\phi}{ds} = \vec{\nabla}\phi \cdot \frac{d\vec{r}}{ds}$, where symbols are used in conventional meaning. Explain the terms present in the right-hand side of the expression. 2

(b) Let R be the distance from a fixed point $\vec{A}(a, b, c)$ to any point $P(x, y, z)$. Show that $\vec{\nabla}R$ is a unit vector in the direction $\overrightarrow{AP} = \vec{R}$. 3

(c) If $\vec{B}(r)$ is both irrotational and solenoidal, show that for a constant vector $\vec{m}$

$$\vec{\nabla} \times (\vec{B} \times \vec{m}) = \vec{\nabla}(\vec{B} \cdot \vec{m})$$ 5

GROUP—B

(Mechanics)

(*Marks* : 40)

6. (*a*) Can a frame of reference be the source of force? Explain. 1

(*b*) Observing a vector $\vec{A}$ from a rotating frame of reference, write its total time derivative. 1

(*c*) State the property of time on which the conservation of mechanical energy rests. 1

(*d*) Is the centre of mass frame of reference an inertial frame? Explain. 1

(*e*) What is principal moment of inertia of a rigid body? 1

(*f*) Due to Tsunami, the duration of the day and night of the earth is changed. Give a simplest explanation of this effect in terms of moment of inertia. 1

7. (*a*) Give schematic diagrams of the two particles collision in laboratory frame and centre of mass frame. 2

(*b*) The force $\vec{F} = (2xy + z^2)\hat{i} + x^2\hat{j} + 2xz\hat{k}$ is a conservative force. Find its potential function. 2

8. Answer any *two* questions : 5×2=10

(a) Find the kinetic energy of a system in its centre of mass frame.

(b) How can you compute the mass of a planet that has a satellite involving the time period of the satellite?

(c) Two particles having masses m_1 and m_2 travel along the x-axis with speeds u_1 and u_2 respectively. After collision their speeds become v_1 and v_2. Prove that the velocities of the centre of mass before and after collision remain same.

9. Answer any *two* questions : 10×2=20

(a) Establish the mathematical expression of acceleration of a particle observed in inertial frame relating the same acceleration observed in rotating frame of reference.

A satellite is moving in a circular polar orbit of radius R with uniform angular velocity ω. As the satellite moves towards the equator, it is observed by radar station situated at latitude λ north of equator. If earth rotates west to east at angular velocity Ω, find the velocity-expression of the satellite obtained by the radar station. 7+3

(b) (i) Show that the momentum of a system in centre of mass frame is always zero.

(ii) Show that the relationship between the angular momentum relative to the centre of mass frame of reference of a system of particles and the angular momentum relative to the laboratory frame is

$$\vec{L} = \vec{L}_{CM} + \vec{r}_{CM} \times \vec{P}$$

4+6

(c) (i) Assuming the earth as spherical, find the expression of its moment of inertia about its axis of symmetry.

(ii) If n and $(n+1)$ be the number of oscillations made by the standard and Kater's reversible pendulum respectively between two consecutive coincidences, then their respective time periods T_0 and T are related by the expression

$$T_0 n = T(n+1)$$

Show that if n is sufficiently large for a second pendulum (that is, $T_0 = 2$ seconds)

$$T = 2\left(1 - \frac{1}{n}\right)$$

7+3

★★★