

2016

MATHEMATICS

( Major )

Paper : 6.4

( Discrete Mathematics )

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. Answer the following as directed : 1×7=7
- (a) State Peano's axioms.
  - (b) Show that the integer 1999 is prime.
  - (c) If  $a|bc$  and  $\gcd(a, b) = 1$ ,  $a, b, c \in \mathbb{Z}$ , then show that  $a|c$ .
  - (d) State Chinese remainder theorem.
  - (e)  $5^{10} - 3^{10}$  is divisible by 11. Justify whether it is true or false.
  - (f) What is the remainder when  $7^{30}$  is divided by 4?
  - (g) If  $\gcd(x, y) = 1$  and  $xy = d^2$ , then  $x$  and  $y$  are also squares. Justify whether it is true or false.

2. Answer the following questions : 2×4=8

(a) If  $a, b, c$  are positive integers such that  $\gcd(a, b) = 1 = \gcd(a, c)$ , then prove that  $\gcd(a, bc) = 1$ .

(b) Show that every square number is of the form  $5k-1, 5k, 5k+1$ , where  $k$  is some positive integers.

(c) Find all solutions of the Diophantine equation  $3x + 2y = 6$ .

(d) If  $p$  is a prime of the form  $4k+1$ , show that there exists a solution in integers  $x, y, m$  of  $x^2 + y^2 = mp$ , where  $0 < m < p$ .

3. Answer the following questions : 5×3=15

(a) Prove that every integer  $n \geq 2$  has a prime factor.

Or

Prove that there are infinitely many primes of the form  $4n+3$ .

(b) If  $p$  is a prime, then show that

$$2\{(p-3)!\} + 1 \equiv 0 \pmod{p}$$

Or

Solve the congruence :

$$72x \equiv 18 \pmod{42}$$

- (c) Prove that if an odd prime number  $p$  can be written as a sum of two squares, then  $p \equiv 1 \pmod{4}$ .

4. (a) Answer either (i) or (ii) : 10

- (i) (1) If  $n$  is a positive integer and  $\gcd(a, n) = 1$ , then show that

$$a^{\phi(n)} \equiv 1 \pmod{n} \quad 5$$

- (2) If  $n = p_1^{k_1} p_2^{k_2} \dots p_t^{k_t}$  is the prime factorization of  $n > 1$ , then show that

$$\sigma(n) = \prod_{i=1}^t \frac{p_i^{k_i+1} - 1}{p_i - 1}$$

Hence show that  $\sigma$  is a multiplicative function. 3+2=5

- (ii) (1) Show that if  $p$  and  $q$  are distinct primes, then

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq} \quad 7$$

- (2) Prove that if 5 divides  $n$ , then

$$\phi(5n) = 5\phi(n) \quad 3$$

(b) Answer either (i) or (ii) : 10

(i) (1) If  $A$  is a tautology and  $A \rightarrow B$ , then show that  $B$  is also a tautology. Examine if the following are adequate systems of connectives : 5

(i)  $(\sim, \wedge)$

(ii)  $(\vee, \rightarrow)$

(2) Let  $p$  stand for ' $\pi$  is a rational number',  $q$  for 'a triangle has two sides' and  $r$  for 'the earth revolves around the sun'. Then find the truth value of the following statement forms : 5

(i)  $p \vee (\sim q \wedge r)$

(ii)  $(p \wedge q) \rightarrow r$

(iii)  $((\sim p) \vee (\sim q)) \leftrightarrow (q \rightarrow r)$

(ii) Prove that the collection  $(\sim, \wedge, \vee)$  is an adequate system of connectives. 10

(c) Answer either (i) or (ii) : 10

(i) (1) What is meant by a complete disjunctive normal form (DNF)? Give one example of a complete DNF. Show that a complete DNF is identically 1. 5

(2) Express the following Boolean expression in disjunctive normal form and conjunctive normal form in the variables present in the expression

$$(xy' + xz)' + x' \quad 5$$

(ii) A committee consisting of three members approves any proposal by majority vote. Each member can approve a proposal by pressing a button attached to their seats. Design as simple a circuit as you can which will allow current to pass when and only when a proposal is approved. 10

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