2016

MATHEMATICS

(Major)

Paper : 6.2

(Numerical Analysis)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) What is meant by normalized floating point representation of real numbers?

(b) Subtract 203.176 from 791.23 in normalized floating point representation.

(c) Define relative error.

(d) Define shift operator E.

(e) Establish the relation $E = 1 + \Delta$.

(f) Evaluate $\Delta^2 (3e^x)$.

(g) What is meant by extrapolation?

2. Answer the following questions : 2×4=8

(a) What assumptions are to be kept in mind for interpolation?

(b) Show that $(1+\Delta)(1-\nabla)=1$, where the symbols have their usual meanings.

(c) Using normalized floating point representation, add $\cdot 4546\ E5$ and $\cdot 5433\ E7$.

(d) Find $f(6)$; it is given $f(0)=-3$, $f(1)=6$, $f(2)=8$, $f(3)=12$, the third difference being constant.

3. Answer the following questions : 5×3=15

(a) Derive Newton's forward interpolation formula.

(b) Use the method of separation of symbols to prove the following identity :

$$u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \cdots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n}$$

Or

Given

$\sin 45° = 0{\cdot}7071$, $\sin 50° = 0{\cdot}7660$

$\sin 55° = 0{\cdot}8192$, $\sin 60° = 0{\cdot}8660$

Find $\sin 52°$ by using any method of interpolation. Mention why you have chosen the particular method.

(Continued)

(c) Find the real root of the equation $x^2 - 5x + 2 = 0$ correct to four places of decimal by using Newton-Raphson method.

4. Answer *either (a) or (b)* :

(a) (i) Derive Simpson's one-third rule. 5

(ii) Calculate an approximate value of

$$\int_0^{\pi/2} \sin x \, dx$$

by Simpson's rule using 11 ordinates. 5

(b) (i) Derive Euler-Maclaurin summation formula. 5

(ii) Find the sum of

$$\frac{1}{51^2} + \frac{1}{53^2} + \frac{1}{55^2} + \cdots + \frac{1}{99^2}$$ 5

5. Answer *either (a) or (b)* :

(a) Explain briefly the idea of numerical integration. Establish the general quadrature formula and deduce trapezoidal rule from it. 2+5+3=10

(b) Derive Stirling's formula. Use Stirling's formula to find y_{28}, given

$$y_{20} = 49225,\ y_{25} = 48316,\ y_{30} = 47236,$$

$$y_{35} = 45926,\ y_{40} = 44306$$

5+5=10

6. Answer *either (a) or (b)* :

(a) *(i)* Give the geometrical interpretation of Newton-Raphson method. 2

(ii) Prove that the bisection method always converges. 3

(iii) Discuss the secant method with appropriate diagram. 5

(b) *(i)* Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position, correct to three decimal places. 5

(ii) Find the first, second and third derivatives of the function tabulated below at the point $x = 1{\cdot}5$: 5

x	1·5	2·0	2·5	3·0	3·5	4·0
$f(x)$	3·375	7·000	13·625	24·000	38·875	59·000

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