

2016

MATHEMATICS

( Major )

Paper : 5.3

( Spherical Trigonometry and Astronomy )

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. Answer all the questions : 1×7=7
- (a) How many great circles can be drawn through two given points?
  - (b) Define spherical triangle.
  - (c) Explain what is meant by 'parallel of latitudes'.
  - (d) At the position of transit of a star, what is the property of altitude and its zenith?
  - (e) What is the relation between the linear velocity and perpendicular distance from the centre upon the tangent to the path of a central orbit?

- (f) Stating the physical situation, define annular solar eclipse.
- (g) State the position of the sun which is known as summer solstice.

2. Answer *all* the following questions :  $2 \times 4 = 8$

- (a) What are meant by ecliptic limits? Explain.
- (b) Show that the sum of the three sides of a spherical triangle is less than the circumference of a great circle.
- (c) Prove that for a right spherical triangle where  $C = \pi/2$ ,  $\cos A = \tan b \cot c$ .
- (d) If  $T$  is the orbital period of a planet, show that a small increment  $\Delta a$  in the semi-axis  $a$  will produce an increase  $\frac{3T\Delta a}{2a}$  in the period.

3. Answer any *three* parts of the following :

$5 \times 3 = 15$

- (a) Explain about the dynamical significance of the Kepler's laws.
- (b) Show that the velocity of a planet in its orbit has got two constant components, one perpendicular to the radius vector and the other perpendicular to the major axis.

- (c) Distinguish between geocentric parallax and annual parallax of a star. Determine the effects of annual or stellar parallax on right ascension and declination.
- (d) Explain with the help of neat diagrams the coordinate systems of celestial sphere.
- (e) If  $a$  is the sun's altitude in the prime vertical at a place of latitude  $\phi$  and  $L$  is its longitude, prove that

$$\phi = \sin^{-1}(\sin L \sin \epsilon \operatorname{cosec} a)$$

4. In a spherical triangle, prove that

$$\cos a \cos C = \sin a \cot b - \sin C \cot B$$

Also prove that, if  $a$  be the side of an equilateral spherical triangle and  $a'$  that of its polar triangle, then

$$2 \cos \frac{a}{2} \cos \frac{a'}{2} = 1 \quad 6+4=10$$

5. Show that the mathematical condition for lunar eclipse to be possible of some kind is

$$\xi < D(1 - 2q \cos i + q^2)^{1/2} \operatorname{cosec} i$$

where  $D = \alpha \pm \gamma_c$  for partial and total eclipse respectively,

$$q = \frac{\theta}{\phi} = \frac{\text{rate of increase of sun's longitude}}{\text{moon's angular velocity in its orbit}}$$

the other symbols carry their usual meanings. 10

6. Discuss the effects of refraction on sunrise and sunset.

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Or

- (a) Show that the retardation due to parallax in the time of rising of an object of geocentric parallax  $p$  seconds of arc and of declination  $\delta$  is

$$\frac{1}{15} \frac{p}{\sqrt{(\cos^2 \phi - \sin^2 \delta)}} \text{ seconds,}$$

$\phi$  being the latitude of the place.

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- (b) If  $S$  is the semi-vertical angle of the tangent cone to the moon from the earth's centre when the moon's horizontal parallax is  $E$  and if  $S', P'$  be another similar parts, prove that the earth being supposed spherical

$$\frac{\sin S}{\sin S'} = \frac{\sin P}{\sin P'}$$

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