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MATHEMATICS

(Major)

Paper : 2·1

(Coordinate Geometry)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×10=10

(a) What is the locus represented by the equation $x^2 - 5xy + 6y^2 = 0$?

(b) What will be the equation of the line $x + y = 2$, when the origin is transferred to the point (1, 1)?

(c) About which axis the parabola $y^2 = 4ax$ is symmetric?

(d) The parametric equations $x = a \sec \phi$ and $y = b \tan \phi$ represent (i) an ellipse, (ii) a parabola, (iii) a hyperbola.

Find the correct answer.

- (e) What are the direction ratios of the normal to the plane $ax + by + cz + d = 0$?
- (f) Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$
- (g) What is the general equation of a cone passing through the coordinate axes?
- (h) Define skew lines.
- (i) Write down the equation of the tangent to $\frac{l}{r} = 1 + e \cos \theta$ at α .
- (j) The shortest distance between two lines is given to be zero. What conclusion can you make about the lines?

2. Answer the following :

2×5=10

- (a) Transform the equation $x^2 - y^2 = a^2$ by taking the perpendicular lines $y - x = 0$ and $y + x = 0$ as coordinate axes.
- (b) Find the equation of the plane through the point $(2, 3, 5)$ and parallel to the plane $2x - 4y + 3z = 9$.
- (c) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are the extremities of any focal chord of the parabola $y^2 = 4ax$, then prove that

$$t_1 t_2 = -1$$

- (d) Find the equation of the plane containing the lines

$$2x + 3y + 5z - 7 = 0$$

$$3x - 4y + z + 14 = 0$$

and passing through the origin.

- (e) Find the equation of the right circular cone whose vertex is the origin, axis is the z-axis and semi vertical angle is α .

3. Answer any two parts : 5×2=10

- (a) If by a rotation of the rectangular axes about the origin, the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then show that

$$a + b = a' + b'$$

$$ab - h^2 = a'b' - h'^2$$

- (b) Prove that the straight lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

will be equidistant from the origin if

$$f^4 - g^4 = c(bf^2 - ag^2)$$

- (c) Find the condition under which the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight lines.

4. Answer any *two* parts : 5×2=10

(a) Find the equation of the pair of tangents from (x', y') to the parabola $y^2 = 4ax$.

(b) Prove that the middle points of the chords of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

parallel to the diameter $y = mx$ lie on the diameter $a^2my = b^2x$.

(c) Prove that the equation of the polar of the origin with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

is $gx + fy + c = 0$.

5. Answer any *four* parts : 5×4=20

(a) Reduce the equation

$$7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$$

to the standard form.

(b) Prove that the length of the focal chord of the conic

$$\frac{l}{r} = 1 - e \cos \theta$$

which is inclined to the axis at an

angle α is $\frac{2l}{1 - e^2 \cos^2 \alpha}$.

- (c) Find the locus of a point such that the sum of the squares of its distances from the planes

$$x+y+z=0, x-z=0, x-2y+z=0$$

is 9.

- (d) Find the shortest distance between the lines

$$ax+by+cz+d=0 = a'x+b'y+c'z+d'$$

and the axis of z .

- (e) A plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point (p, q, r) . Show that the equation of the plane is

$$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$$

- (f) Prove that the equation of the plane through the lines

$$x+y-2z+4=0 = 3x-y+2z+1$$

and parallel to the line

$$\frac{x+2}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$$

is $20x-8y+16z+3=0$.

6. Answer any four parts :

5×4=20

(a) Find the centre and radius of the circle

$$\begin{aligned}x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 &= 0 \\x - 2y + 2z &= 3\end{aligned}$$

(b) Show that the equation of the cylinder whose generators are parallel to the line

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$$

and guiding curve is

$$x^2 + 2y^2 = 1, z = 3$$

is

$$3(x^2 + 2y^2 + z^2) + 8yz - 2zx + 6x - 24y - 18z + 24 = 0$$

(c) Find the condition that the plane $lx + my + nz = p$ may be a tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$.

(d) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes in A, B, C . Prove that the equation of the cone generated by the lines drawn from O to meet the circle ABC is

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$

(7)

(e) Find the equation of the director sphere of the conicoid $ax^2 + by^2 + cz^2 = 1$.

(f) Show that any normal to the conicoid

$$\frac{x^2}{pa+q} + \frac{y^2}{pb+q} + \frac{z^2}{pc+q} = 1$$

is perpendicular to its polar line with respect to the conicoid

$$\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 1$$

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