2016

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×10=10

(a) Write down the nth derivative of $\cos(2x+3)$.

(b) If $z = x^3 y^5 \phi(x/y)$, find the value of

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$$

(c) Find the expression for the subnormal to the curve $y^2 = 4ax$ at any point $P(x, y)$ on the curve.

(d) Write down the radius of curvature for the curve $s = c\tan\psi$.

4. Answer *either* (a) *or* (b) :

(a) (i) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, $|x|<1$, show that

$$(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n = 0$$ 6

(ii) If $y = x^{n-1} \log x$, show that $y_n = \frac{(n-1)!}{x}$. 4

(b) (i) If u is a homogeneous function of x and y of degree n, having continuous partial derivatives, prove that

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^2 u = n(n-1)u$$ 6

(ii) If $v = \sin^{-1} \frac{x^2+y^2}{x+y}$, then show that

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = \tan v$$ 4

5. Answer *either* (a) *or* (b) :

(a) (i) Find the asymptotes of the curve $x^4 - x^2y^2 + x^2 + y^2 - a^2 = 0$. 5

(ii) Show that for the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

the radius of curvature at an extremity of the major axis is equal to half the latus rectum. 5

(b) Define cusp, isolated points, single cusp and double cusp. Find the position and nature of the multiple points on the curve $x^3+2x^2+2xy-y^2+5x-2y=0$. 4+6

6. *(a)* If $U_n=\int_0^{\pi/2} x^n \sin x\, dx\ (n\geq 1)$, show that

$$U_n=n\left(\frac{\pi}{2}\right)^{n-1}-n(n-1)u_{n-2}$$ 5

(b) If $J_n=\int (a^2+x^2)^{n/2}\, dx$, show that

$$J_n=\frac{x(a^2+x^2)^{n/2}}{n+1}+\frac{na^2}{n+1}J_{n-2}$$ 5

7. *(a)* Show that the area enclosed by the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ is $\frac{3}{8}\pi a^2$. 5

(b) Find the surface area of the solid generated by revolving the cardioid $r=a(1+\cos\theta)$ about the initial line. 5

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