2015

MATHEMATICS

(Major)

Paper : 5.4

(Rigid Dynamics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the moment of inertia of a rod of length $2a$ and mass M about a line through its centre perpendicular to its length.

(b) Define equimomental bodies.

(c) Write down the radius of gyration of a circular disc of radius a and mass M about its diameter.

(d) State d'Alembert's principle.

(e) Define compound pendulum.

(f) A particle moves on a plane. What is the degree of freedom of the particle?

(g) Define holonomic system.

2. Answer the following questions : $2\times4=8$

(a) Find the number of degree of freedom for a rigid body which has one point fixed but can move in space about this point.

(b) A rigid body consists of 3 particles of masses 2, 3 and 4 located at (1, −1, 1), (2, 0, 2) and (−1, 1, 0) respectively. Find the moments of inertia about x and y axes.

(c) Give a set of generalized coordinates needed to completely specify the motion of a particle constrained to move on an ellipse.

(d) If a rigid body rotates with angular velocity $\vec{\omega}$ and has angular momentum $\vec{\Omega}$, prove that the kinetic energy is given by

$$T=\frac{1}{2}\vec{\omega}\cdot\vec{\Omega}$$

3. Answer the following questions : $5\times3=15$

(a) Show that the moment of inertia of an ellipse of mass M and semi-axes a and b about a tangent is $\frac{5\,Mp^2}{4}$, where p is the perpendicular from the centre on the tangent.

Or

Find the moment of inertia of a uniform disc of radius a and mass M about a tangent line in its plane.

(b) A uniform circular disc of radius r is oscillating as a compound pendulum in a vertical plane about an axis through a point of the disc perpendicular to the plane and the length of the equivalent simple pendulum is $2r$. If h be the distance of the axis from the centre of the disc, show that

$$h : r = \sqrt{2} - 1 : \sqrt{2}$$

Or

Prove in the case of a compound pendulum that the centres of suspension and oscillation are interchangeable.

(c) Obtain the Lagrange's equations for a conservative holonomic system.

Or

Use Lagrange's equations to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis.

4. AB and AC are two equal similar rods freely hinged at B and lie in a straight line on a smooth table. The end A is struck by a blow perpendicular to AB, show that the resulting velocity of AB is $3\frac{1}{2}$ times that of B. 10

Or

A homogeneous sphere of radius a, rotating with angular velocity ω about a horizontal diameter, is gently placed on a table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time $2\omega a / 7\mu g$ and that then the sphere will roll with angular velocity $\frac{2\omega}{7}$.

5. A rod of length $2a$ is suspended by a string of length l attached to one end. If the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that

$$\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}$$ 10

Or

Show that the kinetic energy of a rigid body moving in any manner is at any instant equal to the kinetic energy of the whole mass, supposed collected at its centre of inertia and moving with it, together with the kinetic energy of the whole mass relative to its centre of inertia.

6. Answer the following questions : 5+5=10

(a) Show that the equation of the momental ellipsoid at the corner of a cube of side $2a$ referred to its principal axis is

$$2x^2 + 11(y^2 + z^2) = \text{constant}$$

(b) Show that the rotational motion of a rigid body is independent of the translatory motion of the body.

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