2015

MATHEMATICS

(Major)

Paper : 5.3

(Spherical Trigonometry and Astronomy)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer *all* the questions : 1×7=7

(a) Define celestial horizon of celestial sphere and name the poles of it.

(b) What do you mean by great circle and what is its connection in determining celestial coordinates?

(c) Before postulating the Kepler's planetary laws, who was the immediate predecessor of Kepler?

(d) Mention a similar property of a small circle and a great circle for a sphere.

(e) Define the polar triangle of a spherical triangle.

(f) What is the difference in totality of lunar and solar eclipses?

(g) State the Cassini's hypothesis of refraction.

2. Answer the following questions : 2×4=8

(a) Mention the physical situations when a lunar eclipse can occur.

(b) Show that in a spherical triangle

$$\pi < A + B + C < 3\pi$$

(c) Show that the zenith distance of a star is the compliment of the altitude.

(d) Defining sidereal time and solar time, distinguish them clearly.

3. Answer any *three* of the following : 5×3=15

(a) If V_1 and V_2 are the linear velocities of a planet at perihelion and aphelion respectively, prove that

$$V_1 : V_2 = (1 + e) : (1 - e)$$

(b) If in a spherical triangle ABC, $A = a$, show that B and b are equal or supplemental.

(c) Show that the right ascension (RA) α and declination δ of the sun is connected by the relation

$$\tan\delta = \tan\varepsilon \sin\alpha$$

(d) Prove that if at a certain instant the declination of a star is unaffected by refraction, the azimuth of the star is then maximum and the star culminates between the pole and the zenith.

4. In any spherical triangle, prove that

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

Hence prove that

$$\frac{\sin(A+B)}{\sin C} = \frac{\cos a + \cos b}{1+\cos c}$$

6+4=10

5. Define true, mean and eccentric anomalies for planet's motion. Deduce the Kepler's equation $m = \phi - e\sin\phi$, where the symbols have their usual meanings. 3+7=10

Or

Describe the situation when a solar eclipse can occur. Prove that at the instant of conjunction in right ascension, the ratio of the distances of the sun from the moon and the earth is

$$\frac{\{\sin P_1 - \sin P \cos(\delta - \delta_1)\}}{\sin P_1}$$

where δ and δ_1 are the declinations of the sun and the moon, P and P_1 are their horizontal parallaxes (the square of $\sin P$ is neglected). 3+7=10

6. *(a)* Show that the refraction of the zenith distance of a star is $k \tan z$, where z is the apparent zenith distance and k is a constant. 5

(b) Show that the parallax in declination of a planet observed from a place in latitude ϕ vanishes if $\tan \phi = \cos H \tan \delta$, δ and H being the planet's declination and hour-angle respectively, and the earth being assumed spherical. 5

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