2015

MATHEMATICS

(Major)

Paper : 5.5

(Probability)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : $1 \times 7 = 7$

(a) Write down the expression for the events A, B and C when only A occurs.

(b) Define probability density function for a continuous random variable.

(c) A random variable X is called discrete if the range of X is

(i) countable

(ii) uncountable

(iii) finite

(Choose the correct answer)

(d) If X and Y are two random variables and

$$\text{var}(X - Y) \neq \text{var}(X) - \text{var}(Y)$$

then what is the relation between X and Y?

(e) The variance of the mean of a random sample is

(i) $\frac{\sigma^2}{n}$

(ii) σ^2

(iii) $\sigma^2 n$

(Choose the correct answer)

(f) If X and Y are two independent Poisson variates, then XY is a _____ variate.

(Fill in the blank)

(g) Under what condition the binomial distribution becomes the normal distribution?

2. Answer the following questions : 2×4=8

(a) If the events A and B are independent of A and B separately, is it necessary that they are independent of $A \cap B$? Justify.

(b) Find K, such that the function f defined by

$$f(x) = Kx^2 \quad \text{when} \quad 0 < x < 1$$
$$= 0 \quad \text{otherwise}$$

is a probability density function. Also determine $P\left(\frac{1}{3} < x < \frac{1}{2}\right)$.

(c) A box contains a white and b black balls. c balls are drawn at random. Find the expectation of the number of white balls drawn.

(d) If X is a random Poisson variate with parameter m, then show that

$$P(X \geq n) - P(X \geq n+1) = (e^{-m} m^n) / \underline{n}$$

3. Answer any *three* of the following : 5×3=15

(a) If $A_1, A_2, \cdots, A_n$ are n mutually exclusive and exhaustive events, then for any event A, prove that

(i) $$P(A) = \sum_{i=1}^{n} P(A_i)\, P(A / A_i)$$

(ii) $$P(A_i / A) = \frac{P(A_i)P(A / A_i)}{P(A)}$$

(b) Define covariance. Prove that the covariance of two independent random variables is zero. Is the converse true? Justify.

(c) Two random variables X and Y are jointly distributed as follows :

$$f(x, y) = \frac{2}{\pi}(1 - x^2 - y^2);\ 0 < x^2 + y^2 < 1$$

Find the marginal distribution of X.

(d) With usual notation for a binomial variate X, given that

$$9P(X = 4) = P(X = 2) \text{ when } n = 6$$

Find the values of p and q.

4. Answer any *three* parts of the following : 10×3=30

(a) (i) The probabilities of n independent events are $p_1, p_2, \cdots, p_n$. Find the expression for the probability that at least one of the events will happen. 5

(ii) In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total products respectively. Of these outputs 5%, 4% and 2% are defective bolts. A bolt is drawn from the product and found to be defective. What is the probability that it is produced by machine A? 5

(b) (i) Two random variables X and Y have the following joint probability distribution function :

$$f(x, y) = 2 - x - y; \quad 0 \le x \le 1, 0 \le y \le 1$$

$$= 0 \quad \text{otherwise}$$

Find—

(1) marginal density function;

(2) $E(X)$ and $E(Y)$;

(3) conditional density function. 5

(ii) A function $f(x)$ of x is defined as follows :

$$f(x) = 0 \quad \text{for} \quad x < 2$$

$$= \frac{1}{18}(3+2x) \quad \text{for} \quad 2 \le x \le 4$$

$$= 0 \quad \text{for} \quad x > 4$$

Show that it is a density function. Also find the probability that a variate with this density will lie in the interval $2 \le x \le 3$. 5

(c) (i) Prove that the variance of a random variable X can be expressed as the sum of the expectation of the conditional variance and the variance of the conditional expectation, i.e.,

$$\text{var}(X) = E[\text{var}(X|Y)] + \text{var}[E(X|Y)] \quad 5$$

(ii) Let the random variable X_i assume values i and $-i$ with equal probabilities. Show that the law of large number cannot be applied to the independent variables $X_1, X_2, \cdots, X_n$. 5

(d) (i) If X is a binomial variate, then prove that

$$\text{cov}\left(\frac{X}{n}, \frac{n-X}{n}\right) = -\frac{pq}{n}$$

where p and q have their usual meanings. 5

(ii) If 3% of electric bulbs manufactured by a company are defective, using Poisson's distribution, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective.

[Given $e^{-3} = 0 \cdot 04979$] 5

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