

Total No. of printed pages = 7

3 (Sem 4) MAT M2

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MATHEMATICS

(Major)

Paper : M-4.2

Full Marks -80

Time - Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following question : $1 \times 10 = 10$
- (a) What is the physical significance of the moment of a force ?
 - (b) Under what conditions the effect of a couple is not altered if it is transferred to a parallel plane ?
 - (c) Define Angle of friction.
 - (d) What is the position of C. G. of a uniform parallelogram lamina ?

[Turn over

(e) When a body is said to be in unstable equilibrium ?

(f) What is Poinsot's Central axis ?

(g) State the principle of virtual work.

(h) Define amplitude and frequency of a Simple Harmonic Motion.

(i) What are the characteristics of a central orbit ?

(j) What do you mean by terminal velocity ?

2. Answer the following questions : $2 \times 5 = 10$

(a) Prove that a single force and a couple in the same plane are equivalent to a single force, equal and parallel to the given single force.

(b) The algebraic sum of moments of a system of forces about four points whose coordinates are (x_1, y_1) , (x_2, y_2) , (x_3, y_3) and (x_4, y_4) referred to \overline{OX} , \overline{OY} as rectangular axes are G_1 , G_2 , G_3 , G_4 respectively. Show that

$$\begin{vmatrix} 1 & x_1 & y_1 & G_1 \\ 1 & x_2 & y_2 & G_2 \\ 1 & x_3 & y_3 & G_3 \\ 1 & x_4 & y_4 & G_4 \end{vmatrix} = 0$$

- (c) State the laws of Friction.
- (d) The speed v of a particle moving along x axis is given by the relation $v^2 = n^2 (8bx - x^2 - 12b^2)$. Prove that the motion is Simple Harmonic.
- (e) Find the law of force towards the pole under which the curve $au = e^{n\theta}$ is described.
(The symbols have their usual meanings).

3. Answer the following questions : $4 \times 5 = 20$

(a) Prove that any system of coplanar forces acting on a rigid body can ultimately be reduced to a single force acting at any arbitrarily chosen point in the plane, together with a couple.

(b) A uniform ladder is in equilibrium with one end resting on the ground and other against a vertical wall. If the ground and wall be both rough, the coefficient of friction being μ and μ' respectively, and if the ladder be on the point of slipping at both ends, then show that the inclination of the ladder to the horizon is

given by $\tan\theta = \frac{1 - \mu\mu'}{2\mu}$.

- (c) Find the C.G. of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ lying in the first quadrant.

Or

Find the C.G of the surface formed by the revolution of the cardioid $r = a(1 + \cos\theta)$ about its axis.

- (d) A body moving in a straight line OAB with Simple Harmonic motion has zero velocity at the points A and B whose distances from O are a and b respectively and has velocity V when half way between them. Show that the complete period is $\frac{\pi(b-a)}{V}$.

- (c) If v_1 and v_2 are the linear velocities of a planet when it is respectively nearest and farthest from the sun, prove that $(1-e)v_1 = (1+e)v_2$.

4. Answer the following questions : 5×4=20

- (a) State and prove the necessary and sufficient conditions that a system of coplanar forces acting on a rigid body may be in equilibrium.
- (b) Write down the forces which may be omitted in forming the equation of virtual work of a system of coplanar forces acting at different points of a rigid body.

- (c) A point moving in a straight line with Simple Harmonic Motion has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2 . Show that the period of motion is

$$2\pi \sqrt{\left(\frac{x_1^2 - x_2^2}{v_1^2 - v_2^2} \right)}$$

- (d) A particle is projected vertically upwards under gravity, supposed constant, in a resisting medium whose resistance varies as the square of the velocity. Show that the velocity of the particle in any position is given by

$$x = \frac{V^2}{2g} \log \frac{V^2 + u^2}{V^2 + v^2}$$

5. Answer the following :

- (a) Obtain the differential equation of the path of a particle moving in a plane with an acceleration which is always directed to a fixed point in a plane. 5
- (b) One end of an elastic string (modulus of elasticity λ) whose natural length is a , is fixed to a point on a smooth horizontal table and the other is tied to a particle of mass m , which is lying on the table. The particle is

pulled to a distance from the point of attachment of the string equal to twice its natural length and then let go. Show that the

time of complete oscillation is $2(\pi + 2)\sqrt{\left(\frac{am}{\lambda}\right)}$

5

Or

A particle of mass m is travelling along x -axis such that at $t = 0$, it is located at $x = 0$ and has speed v_0 . The particle is acted upon by a force which opposes the motion and has magnitude proportional to the instantaneous speed. Find the speed, position and acceleration of the particle at any time $t(>0)$.

6. Answer the following :

(a) Three forces act along the straight lines $x = 0$, $y - z = a$; $y = 0$, $z - x = a$; $z = 0$, $x - y = a$. Show that they cannot reduce to a couple. 5

(b) A body rests in equilibrium upon another fixed body, the portions of the two bodies in contact have radii of curvature ρ_1 and ρ_2 respectively. The centre of gravity of the first body is at a height h above the point of

contact and the common normal makes an angle α with the vertical. Prove that the equilibrium is stable or unstable according as

$$h < \text{or} > \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \cos \alpha$$

5

Or

A square of side : $2a$ is placed with its plane vertical between two smooth pegs which are in the same horizontal line at a distance c apart. Show that it will be in equilibrium when the inclination of one of its edges to the horizon is either 45° or

$$\frac{1}{2} \sin^{-1} \left(\frac{a^2 - c^2}{c^2} \right).$$