

Total No. of printed pages = 8
3 (Sem 3) MAT M2

2015

MATHEMATICS

(Major)

Paper : 3.2

(Linear Algebra and Vector)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks
for the questions.

GROUP – A

(Linear Algebra)

Marks : 40

1. Answer the following as directed : $1 \times 7 = 7$

(a) Show that in a vector space $V(F)$

$$\alpha v = 0, v \neq 0 \Rightarrow \alpha = 0$$

$$\alpha v = 0, \alpha \neq 0 \Rightarrow v = 0, \text{ where } v \in V, \alpha \in F.$$

[Turn over

(b) Let S be the subset of \mathbb{R}^3 defined by

$$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$$

Examine whether S is a subspace of \mathbb{R}^3 .

(c) In \mathbb{R}^3 , $\alpha = (4, 3, 5)$, $\beta = (0, 1, 3)$,
 $\gamma = (2, 1, 1)$.

Is α a linear combination of β and γ ?

(d) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by

$$T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$$

Examine whether T is a linear transformation.

(e) Let T be a linear operator on \mathbb{R}^2 which is represented by the following matrix :

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

with respect to the standard ordered basis.
Then T has no eigen value in \mathbb{R} . -Justify whether it is true or false.

(f) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $T(x, y) = (x, 0)$.
What is the eigen space of T associated with the eigen value 1?

(g) If λ is a simple eigen value (i.e. of multiplicity 1) of an $n \times n$ matrix A , then rank of $(A - \lambda T)$ is

(i) $n+1$

(ii) $n-1$

(iii) n

(iv) none of these

—Choose the correct option.

2. Answer the following questions : $2 \times 4 = 8$

(a) Let S and T be two non-empty finite subsets of a vector space v over a field F and $S \subset T$. Show that $L(S) \subset L(T)$, (where $L(S)$ and $L(T)$ denote the linear spans of S and T respectively).

(b) Let v and u be vector spaces over the same field F and $T : v \rightarrow u$ be a linear transformation. Show that $\ker T = \{0\}$ if and only if T is one-one.

(c) A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is defined by $T(x, y, z) = (3x - 2y + z, x - 3y - 2z)$.

Find the matrix of T relative to the ordered bases $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ of \mathbb{R}^3 and $\{(1, 0), (0, 1)\}$ of \mathbb{R}^2 .

(d) Using Cayley Hamilton theorem, compute the

$$\text{inverse of } A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}.$$

3. Answer any one part :

5

(a) Find the range, rank, kernel and nullity of the following linear transformation :

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ such that}$$

$$T(x, y) = (x + y, x - y, y).$$

(b) Determine the linear transformation

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^2 \text{ which maps the basis vectors}$$

$(1, 0, 0), (0, 1, 0), (0, 0, 1)$ of \mathbb{R}^3 to the vectors $(1, 1), (2, 3)$ and $(3, 2)$ respectively.

Also determine $\ker T$.

4. Answer the following questions : $10 \times 2 = 20$

(a) Let W be a subspace of a finite dimensional vector space V . Then show that W is also finite dimensional and $\dim W \leq \dim V$. Also show that $\dim V = \dim W$ if and only if $V = W$.

Or

Let W be a subspace of a finite dimensional vector space V . Prove that there exists a subspace W' of V such that $V = W \oplus W'$.

- (b) Prove that similar matrices have same characteristic polynomial. Let A be a real $n \times n$ matrix. Let λ be a real eigen value of A . Show that there exists an eigen vector X of A corresponding to eigen value λ such that X is also real.

Or

Obtain the eigen values, eigen vectors and eigen spaces of the matrix :

$$A = \begin{pmatrix} 2 & 2 & -6 \\ 2 & -1 & -3 \\ 2 & -1 & 1 \end{pmatrix}$$

Is A diagonalisable ?

GROUP - B

(Vector)

Marks : 40

5. Answer the following as directed : $1 \times 3 = 3$

(a) If $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$,

$$\vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k},$$

$$\vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k},$$

Find $\vec{a} \times (\vec{b} \times \vec{c})$.

(b) Examine whether $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar.

(c) $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k})$ is equal to

(i) $2\vec{b}$

(ii) $2\vec{a}$

(iii) 0

(iv) none of these

— Choose the correct option.

6. Show that if \vec{a} is perpendicular to both \vec{b} and \vec{c} , then

$$[\vec{a} \vec{b} \vec{c}]^2 = \vec{a}^2 (\vec{b} \times \vec{c})^2 \quad 2$$

7. Answer the following questions : $5 \times 3 = 15$

(a) Show that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ if and only

if either $\vec{b} = 0$, or \vec{c} is collinear with \vec{a} , or \vec{b} is perpendicular to both \vec{a} and \vec{c} .

Or

If $\vec{a} = (1, 1, 1)$, $\vec{b} = (2, -1, 3)$, $\vec{c} = (1, -1, 0)$,

$\vec{d} = (6, 2, 3)$, express \vec{d} in terms of

$\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$.

(b) If $\vec{r} = a \cos t \hat{i} + a \sin t \hat{j} + (a \tan \alpha)t \hat{k}$,

$$\text{find } \left| \frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2} \right|$$

$$\text{and } \left[\frac{d\vec{r}}{dt} \frac{d^2\vec{r}}{dt^2} \frac{d^3\vec{r}}{dt^3} \right]$$

(c) Determine the constant 'a' so that the vector $\vec{f} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + az)\hat{k}$ is solenoidal.

8. Answer the following questions : $10 \times 2 = 20$

(a) A particle moves so that its position vector is given by $\vec{r} = \cos \omega t \hat{i} + \sin \omega t \hat{j}$, where ω is a constant. Show that:

(i) the velocity of the particle is perpendicular to \vec{r}

(ii) the acceleration is directed towards the origin and has magnitude proportional to the distance from the origin, and

(iii) $\vec{r} \times \frac{d\vec{r}}{dt}$ is a constant vector.

3+3+4=10

Or

If \vec{a} is a constant vector, prove that

$$\operatorname{div}(\vec{r}^n(\vec{a} \times \vec{r})) = 0,$$

$$\text{where } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}. \quad 10$$

(b) Evaluate : $\iint_S (y^2 z^2 \hat{i} + z^2 x^2 \hat{j} + x^2 y^2 \hat{k}) \cdot d\vec{S}$,

where S is the part of the sphere

$$x^2 + y^2 + z^2 = 1 \text{ above the } xy\text{-plane.} \quad 10$$

Or

Find the work done when a force

$\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ moves a particle in xy -plane from $(0, 0)$ to $(1, 1)$ along the parabola $y^2 = x$. If C is the circle $x^2 + y^2 = 4, z = 0$, find the circulation of \vec{F} along the curve C .

$$5+5=10$$

Total No. of printed pages = 10
3 (Sem 3) PHY M1

2015

PHYSICS

(Major)

Paper : 3.1

(Mathematical Methods-III and Electrostatics)

Full Marks - 60

Time - Three hours

The figures in the margin indicate full marks for the questions.

GROUP-A

(Mathematical Physics)

Marks : 25

1. Answer the following questions : $1 \times 3 = 3$

(a) Define rank of a matrix.

(b) When a symmetric matrix is said to be Hermitian?

(c) Find adj A if $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(C) (Turn) over

Or

If \vec{a} is a constant vector, prove that

$$\operatorname{div}(\vec{r}^n(\vec{a} \times \vec{r})) = 0,$$

$$\text{where } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

10

(b) Evaluate : $\iint_S (y^2z^2\hat{i} + z^2x^2\hat{j} + x^2y^2\hat{k}) \cdot d\vec{S}$,

where S is the part of the sphere

$$x^2 + y^2 + z^2 = 1 \text{ above the } xy\text{-plane.}$$

10

Or

Find the work done when a force

$\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ moves a particle in xy-plane from (0, 0) to (1, 1) along the parabola $y^2 = x$. If C is the circle $x^2 + y^2 = 4, z = 0$, find the circulation of \vec{F} along the curve C.

5+5=10

Total No. of printed pages = 10
3 (Sem 3) PHY M1

2015

PHYSICS

(Major)

Paper : 3.1

(Mathematical Methods-III and Electrostatics)

Full Marks - 60

Time - Three hours

The figures in the margin indicate full marks for the questions.

GROUP-A

(Mathematical Physics)

Marks : 25

1. Answer the following questions : $1 \times 3 = 3$

(a) Define rank of a matrix.

(b) When a symmetric matrix is said to be Hermitian?

(c) Find $\text{adj } A$ if $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(Turn) over