Total No. of printed pages = 10

3 (Sem 1) MAT M1

2015

MATHEMATICS

(Major)

Paper : 1.1

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer/choose the correct option : 1×10=10

(a) Give an example of a relation on the set of real numbers $\mathbb{R}$ which is reflexive and transitive but not symmetric.

(b) Find all partitions of the set $x = \{1, 2, 3\}$.

(c) Let Q_+ be the set of all positive rational numbers and $*$ be a binary operation on Q_+ defined by $a * b = \frac{ab}{3}$, $\forall$ a, b $\in Q_+$. Find the identity element of Q_+ and determine the inverse of nonzero element $a \in Q_+$.

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(d) Define an operation $*$ on the set of real numbers $\mathbb{R}$ as $a * b = a + 2b$, $\forall\ a, b \in \mathbb{R}$. Then $(\mathbb{R}, *)$ is not a group because

(i) $\mathbb{R}$ is not closed w.r.t $*$

(ii) $\mathbb{R}$ is not associative w.r.t $*$

(iii) Identity element does not exist.

(iv) Inverse of each nonzero element does not exist.

(e) The value of i^i is

(i) $e^{-(4n+1)\frac{\pi}{2}}$ (ii) $e^{(4n-1)\frac{\pi}{2}}$

(iii) $e^{(4n+1)\frac{\pi}{2}}$ (iv) $e^{-(4n-1)\frac{\pi}{2}}$

(f) If z is a complex number, then $\sin^{-1}z$ is

(i) $-i \log\left(iz \pm \sqrt{1-z^2}\right)$

(ii) $i \log\left(iz \pm \sqrt{1+z^2}\right)$

(iii) $-i \log\left(z \pm i\sqrt{1-z^2}\right)$

(iv) $i \log\left(z \pm i\sqrt{1+z^2}\right)$

(g) If A is a square matrix, then adj A'−(adj A)' is

(i) $2\,|A|$ (ii) $2\,|A|\,I$

(iii) null matrix (iv) identity matrix

(h) Inverse of which of the following matrices exists ? Given that 1, ω, ω^2 are the cube roots of unity.

(i) $\begin{pmatrix} 1 & \omega \\ \omega & \omega^2 \end{pmatrix}$

(ii) $\begin{pmatrix} \omega^2 & 1 \\ 1 & \omega \end{pmatrix}$

(iii) $\begin{pmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} \omega & \omega^2 \\ \omega & 1+\omega \end{pmatrix}$

(i) The rank of the matrix

$$\begin{pmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix} \text{ is}$$

(i) 0 (ii) 1

(iii) 2 (iv) 3

(j) If $2+\sqrt{3}$ is a root of the equation $x^4+2x^3-16x^2-22x+7=0$, then the other three roots are

(i) $2-\sqrt{3},\ 3+\sqrt{2},\ -3+\sqrt{2}$

(ii) $-2-\sqrt{3},\ -3+\sqrt{2},\ -3-\sqrt{2}$

(iii) $2-\sqrt{3},\ -3+\sqrt{2},\ -3-\sqrt{2}$

(iv) $-2+\sqrt{3},\ 3+\sqrt{2},\ -3-\sqrt{2}$

2. Give answers to the following questions : 2×5=10

(a) If $f : A \to B$ and $g : B \to C$ are bijective mappings, then prove that gof is also a bijective mapping.

(b) In a group G, prove that $(ab)^{-1} = b^{-1} a^{-1}$, $\forall$ a, b $\in$ G.

(c) Show that every real symmetric matrix is Hermitian.

(d) Prove that

$$\pi = 2\sqrt{3}\left(1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \ldots\ldots\right).$$

(e) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of

$$\sum \frac{1}{\alpha^2\beta^2}.$$

3. Answer any *four* parts : 5×4=20

(a) Define an equivalence relation on a nonempty set.

Show that the relation "congruence modulo m" is an equivalence relation on the set of integers. 1+4=5

(b) Let $f : A \to B$, $g : B \to C$, $h : C \to D$ be three mappings. Prove that

(i) $ho\,(gof) = (hog)of$

(ii) $foi = f$ and $jof = f$ where $i : A \to A$ and $j : B \to B$ are identity mappings. 3+2=5

(c) If G is a group in which $(ab)^i = a^i b^i$ for three consecutive integers i and any a, b in G, then prove that G is abelian. 5

(d) (i) Find the values of $(1+i)^{\frac{1}{3}}$

(ii) Prove that

$$\sin^2\theta\cos\theta = \theta^2 - \frac{5}{6}\theta^4 + \ldots\ldots$$

$$+(-1)^{n+1}\frac{3^{2n}-1}{4(2n)!}\theta^{2n} + \ldots\ldots$$ 2+3=5

(e) (i) Apply Descarte's rule of signs to ascertain the minimum number of complex root of the equation $x^7-3x^3-x+1=0$.

(ii) If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, find the equation whose roots are $\alpha\beta+\beta\gamma$, $\beta\gamma+\gamma\alpha$ and $\gamma\alpha+\alpha\beta$. 3+2=5

(f) Define rank of a matrix. Find the rank of the matrix 1+4=5

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

4. Answer any *one* part : 10

(a) (i) Prove that if G is a finite group and H is a subgroup of G, then 0 (H) divides 0 (G).

(ii) If G is a finite group, show that for any $a \in G$, $a^{0(G)} = e$ where e is the identity element of the group G.

(iii) Prove that every quotient group of a cyclic group is cyclic. 4+2+4=10

(b) (i) Prove that centre of a group G is a subgroup of the group G.

(ii) For any integer a and a prime p, prove that $a^p \equiv a \pmod{p}$.

(iii) Prove that a finite group of prime order is abelian. 3+3+4=10

5. Answer any *one* part : 10

(a) (i) If z is a complex number, show that

$$|z| \geq \frac{1}{\sqrt{2}}(|\text{Rl } z| + |\text{Im } z|).$$

(ii) Express $\log(x + iy)$ in the form $A + iB$ where A and B are reals. Also find $\log(x + iy)$.

(iii) If $\cos^{-1}(\alpha + i\beta) = \theta + i\phi$, prove that $\alpha^2 \sec h^2\phi + \beta^2 \operatorname{cosec} h^2\phi = 1$

3+4+3=10

(b) (i) Two complex numbers z_1 and z_2 are such that $|z_1 + z_2| = |z_1 - z_2|$. Show that amp z_1 and amp z_2 differ by $\frac{\pi}{2}$ or $\frac{3\pi}{2}$.

(ii) Expand $\tan x$ in ascending powers of x.

(iii) Deduce the Gregory series

$$\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5} + \tan^5\theta - \cdots\cdots\infty,$$

where $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$.

3+3+4=10

6. Answer any *two* parts : 5×2=10

(a) Find the value of k and solve the equation $8x^3 - 12x^2 - kx + 3 = 0$ if the roots are in arithmetic progression. 5

(b) Solve the equation $x^3 - x^2 + 3x - 27 = 0$ if it have three distinct roots of equal moduli. 5

(c) Solve the equation $x^4-4x^3-4x^2-4x-5=0$ given that two roots are connected by the relation $2\alpha + \beta = 3$. 5

(d) Solve the equation $x^3-3x-1=0$ by Cardon's method. 5

7. Answer any *two* parts : 5×2=10

(a) Define a symmetric and a skew-symmetric matrices.

Show that the matrix B'AB is symmetric or skew-symmetric according as A is symmetric or skew-symmetric. 2+3=5

(b) If A is a square matrix of order n, prove that

$$|\text{adj}\,(\text{adj}A)| = |A|^{(n-1)^2}$$ 5

(c) Prove that the necessary and sufficient condition for a matrix A to possess an inverse is that $|A| \neq 0$. 5

(d) For what values of λ the equations

$$x+y+z=1$$

$$x+2y+4z=\lambda$$

$$x+4y+10z=\lambda^2$$

have a solution and solve them completely in each case. 5

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3 (Sem 1) MAT M2

2015

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following : $1\times10=10$

(a) Write down the nth derivative of $\log_e(ax+b)$.

(b) If $u=\sin^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}$, find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$.

(c) Find $\frac{ds}{dx}$ for the curve $y^2 = 4ax$.

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