Bijni College Library
P.O. Bijni, Dist. Chirang
(B.T.A.D) Assam

2014

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×10=10

(a) The differential equation

$$\frac{dx}{dt} = \frac{x + 2xt + \cos t}{1 + t^2}$$

is

(i) linear and not separable

(ii) separable and not linear

(iii) both separable and linear

(iv) neither separable nor linear

(Choose the correct answer)

(b) What is the integrating factor of the differential equation

$$\frac{dy}{dx}+\left(\frac{x}{1+x}\right)y=1+x\ ?$$

(c) Write the standard form of homogeneous differential equation of order n.

(d) What do you mean by self-orthogonal family of curves?

(e) Write the particular integral of the differential equation

$$(D-2)^3 y=e^{2x}$$

(f) Write down the general solution of the differential equation

$$y=px+ap(1-p)$$

(g) Give the geometrical interpretation of the differential equation

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$$

(h) Write the conditions for exactness of the differential equation

$$P\,dx+Q\,dy+R\,dz=0$$

(i) The partial differential equations can be formed by the elimination of

(i) arbitrary constants only

(ii) arbitrary functions only

(iii) arbitrary functions or arbitrary constants

(iv) None of the above

(Choose the correct answer)

(j) Find an integral belonging to complementary function of the differential equation

$$(1-x^2)y_2 + xy_1 - y = x(1-x^2)^{3/2}$$

2. Answer the following questions : 2×5=10

(a) Solve :

$$\frac{dy}{dx} = \sec(x+y)$$

(b) Solve :

$$(D^3 - 4D^2 + 5D - 2)y = 0$$

(c) Eliminate the arbitrary functions f and F from $z = f(x+iy) + F(x-iy)$.

(d) Solve :

$$x\,dy - y\,dx - 2x^2 z\,dz = 0$$

(e) Construct the partial differential equation by eliminating a and b from

$$z = ax + by + a^2 + b^2$$

3. Answer any *four* questions : 5×4=20

(a) Prove that a necessary and sufficient condition that the differential equation

$$M\,dx + N\,dy = 0$$

be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(b) Find the orthogonal trajectories of the family of curves $3xy = x^3 - a^3$, a being parameter of the family.

(c) What is a first-order differential equation? Reduce the following differential equation to linear form :

$$\frac{dy}{dx} + Py = Qy^n$$

Also write the integrating factor.

(d) Solve by method of variation of parameter

$$y_2 + n^2 y = \sec nx$$

(e) Solve :

$$z(xp - yq) = y^2 - x^2$$

(f) Explain how to get general solution and singular solution of a differential equation of the form

$$y = px + f(p) \quad \left(p = \frac{dy}{dx} \right)$$

4. Answer *either (a)* and *(b) or (c)* and *(d)* : 5+5=10

(a) Show that the equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through the origin is $y = 2(e^x - x - 1)$.

(b) Solve :

$$(x^3D^3 - x^2D^2 + 2xD - 2)y = x^3 + 3x$$

(c) Solve :

$$(D^2 - 4D + 4)y = e^x + \cos 2x$$

(d) Show that $Ax^2 + By^2 = 1$ is the solution of the differential equation

$$x\left\{ y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \right\} = y\frac{dy}{dx}$$

Also write the order and degree of the differential equation.

5. Answer *either (a)* and *(b)* or *(c)* and *(d)* : 5+5=10

(a) Solve :

$$x^2y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$$

(b) Remove the second term from the following equation and hence solve :

$$\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right)y = xe^x$$

(c) Solve :

$$\cos x \frac{d^2y}{dx^2} + \frac{dy}{dx}\sin x - 2y\cos^3 x = 2\cos^5 x$$

(d) Solve the following equation, given that $\frac{\sin x}{x}$ is a part of complementary function :

$$\frac{d^2y}{dx^2} + \left[1 + \frac{2\cot x}{x} - \frac{2}{x^2}\right]y = x\cos x$$

6. Answer *either (a)* and *(b)* or *(c)* and *(d)* : 5+5=10

(a) Solve :

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$

(b) Find $f(y)$ such that the total differential equation

$$f(y)\,dx - xz\,dy - xy\log y\,dz = 0$$

is integrable. Find the corresponding integral.

(c) Solve :

$$\frac{d^2x}{dt^2} - 3x - 4y = 0;\ \frac{d^2y}{dt^2} + x + y = 0$$

(d) Solve :

$$xz^3\,dx - z\,dy + 2y\,dz = 0$$

7. Answer *either (a) and (b) or (c) and (d)* : 5+5=10

(a) Solve by Lagrange's method

$$(y^2 + z^2 - x^2)p - 2xyq + 2zx = 0$$

(b) Find the integral surface of the linear partial differential equation

$$x(y^2 + z)\,p - y(x^2 + z)\,q = (x^2 - y^2)\,z$$

which contains the straight line $x + y = 0,\ z = 1$.

(c) Solve by Charpit's method

$$pxy + pq + qy = yz$$

(d) Find the complete integral of

$$p^3 + q^3 = 27z$$

Find also the singular integral if it exists.

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