2013

# MATHEMATICS

( Major )

Paper : 2.2

**( Differential Equation )**

*Full Marks* : 80

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

1. Answer the following : 1×10=10

(a) What do you mean by order and degree of a differential equation?

(b) Determine if the following differential equation is homogeneous :

$$y_1 = \frac{x^2 + y}{x^3}$$

(c) Write down the general form of a linear differential equation of order two.

(d) Write down the integrating factor of the differential equation :

$$(1+x^2)y_1 + 2xy - 4x^2 = 0$$

(e) Find the particular integral of the differential equation

$$(D^2 - 4)y = \sin 2x$$

(f) When is the total differential equation

$$Pdx + Qdy + Rdy = 0$$

($P$, $Q$, $R$ are functions of $x$, $y$ and $z$) said to be exact?

(g) Write down the general form of a linear partial differential equation of order one with $n$ independent variables.

(h) Write down the general solution of the differential equation

$$p^2 - 5p + 6 = 0; \; p = \frac{dy}{dx}$$

(i) What do you mean by trajectory of a given family of curves?

(j) What is complementary function of the differential equation

$$(D^2 + 4D + 4)y = x^3 \, ?$$

**2.** Answer the following : 2×5=10

(a) Solve :

$$\frac{xdx}{y^2 z} = \frac{dy}{xz} = \frac{dz}{y^2}$$

(b) Solve :

$$\frac{dy}{dx} = (4x + y + 1)^2$$

(c) Solve :

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$$

(d) Solve :

$$(2x^3 y + 1)dx + x^4 dy + x^2 \tan z dz = 0$$

(e) Construct the partial differential equation by eliminating $a$, $b$ and $c$ from $z = a(x + y) + b(x - y) + abt + c$.

**3.** Answer any *four* parts : 5×4=20

(a) Solve

$$(px - y)(py + x) = h^2 p$$

using the transformation $x^2 = u$ and $y^2 = v$.

(b) Show that the following equation is exact and hence solve it

$$(1+e^{x/y})dx+e^{x/y}(1-\frac{x}{y})dy=0$$

(c) Solve :

$$x^2D^2y-3xDy+5y=x^2\sin\log x$$

(d) Solve by the method of variation of parameter :

$$y_2+4y=4\tan 2x$$

(e) Find the differential equation of all spheres of radius $c$ having centre in the $xy$-plane.

(f) Solve by Lagrange's method :

$$(y^2+z^2-x^2)p-2xyq+2zx=0$$

4. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Show that the system of confocal and co-axial parabolas $y^2=4a(x+a)$ is self-orthogonal, $a$ being parameter.

(b) Solve :

$$\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$$

(c) Solve :

$$(D^3+8)y=x^4+2x+1$$

(d) Show that $Ax^2 + By^2 = 1$ is the solution of

$$x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

**5.** Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Solve :

$$x^2y_2 - (x^2 + 2x)y_1 + (x+2)y = x^3e^x$$

(b) Transform the differential equation

$$\cos xy_2 + \sin xy_1 - 2y\cos^3 x = 2\cos^5 x$$

into one having $z$ as independent variable, where $z = \sin x$ and solve it.

(c) Reduce the differential equation

$$y_2 - 4xy_1 + (4x^2 - 1)y = -3e^{x^2}\sin 2x$$

to its normal form and hence solve it.

(d) Show that if $z$ satisfies

$$\frac{d^2z}{dx^2} + p\frac{dz}{dx} = 0$$

by changing the independent variable $x$ to $z$, we shall transform

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

into

$$\frac{d^2y}{dz^2} + Q_1y = R_1$$

where $P$, $Q$, $R$ are functions of $x$.

**6.** Answer *either* *(a)* and *(b)* *or* *(c)* and *(d)* : 5+5

*(a)* Solve :

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$$

*(b)* Show that the necessary condition for integrability of single differential equation $Pdx + Qdy + Rdz = 0$ is

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

*(c)* Solve :

$$\frac{dx}{dt} - 7x + y = 0$$
$$\frac{dy}{dt} - 2x - 5y = 0$$

*(d)* Find $f(z)$ such that

$$\left(\frac{y^2+z^2-x^2}{2x}\right)dx - ydy + f(z)dz = 0$$

is integrable. Hence solve it.

**7.** Answer *either* *(a)* and *(b)* *or* *(c)* and *(d)* : 5+5

*(a)* Solve by Charpit's method :

$$2xz - px^2 - 2qxy + pq = 0$$

*(b)* Find the integral surface of the linear partial differential equation

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

which contains the straight line $x + y = 0$, $z = 1$.

*(c)* Derive the partial differential equation by the elimination of arbitrary function $\phi$ from the equation

$$\phi(u, v) = 0$$

where $u$ and $v$ are functions of $x$, $y$ and $z$.

*(d)* Find the complete integral of

$$pq = 1$$

Find also the singular integral if it exists.

★ ★ ★