

3 (Sem-2) MAT M 1

Bijni College Library  
2 0 1 3 P.O.-Bijni, Dist.-Chirang  
(B.T.A.D)Assam

MATHEMATICS

( Major )

Paper : 2.1

( Coordinate Geometry )

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. (a) What will be the equation of the line  $x + y = 2$ , when the origin is transferred to the point (1, 1)? 1
- (b) Write down the condition for pair of lines represented by the equation  
 $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  1
- (c) Write down the parametric equations of the parabola,  $y^2 = 4ax$ . 1
- (d) What are the direction ratios of the normal to the plane,  $x + y + z = 0$ ? 1
- (e) Write down the direction cosines of z-axis. 1

Bijl College Library  
 100, Disi Ching  
 (D) Assar

- (f) What conic does the following equation represent? 1

$$x^2 + 2xy + y^2 - 2x - 1 = 0$$

- (g) Write down the equation of the tangent to

$$\frac{l}{r} = 1 + e \cos \theta$$

at  $\alpha$ . 1

- (h) What are the centre and radius of the sphere

$$2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$$
 1

- (i) Define skew lines. 1

- (j) Under what condition, the lines represented by  $ax^2 + 2hxy + by^2 = 0$  will be perpendicular to each other. 1

2. (a) Transform the equation  $x^2 - y^2 = a^2$  by taking the perpendicular lines

$$y - x = 0 \text{ and } y + x = 0$$

as coordinate axes. 2

- (b) Find the equation of the sphere through circles  $x^2 + y^2 + z^2 = 25$ ,  $x + 2y - z + 2 = 0$  and the point  $(1, 1, 1)$ . 2

- (c) The axis of a right circular cylinder is

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$$

and its radius is 5. Find its equation. 2

- (d) If  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$  are the extremities of any focal chord of the parabola,  $y^2 = 4ax$ , prove that

$$t_1 t_2 = -1 \quad 2$$

- (e) Find the equation of the plane containing the lines

$$2x + 3y + 5z - 7 = 0, \quad 3x - 4y + z + 14 = 0$$

and passing through the origin. 2

3. (a) A sphere of constant radius,  $r$  passes through the origin,  $O$  and cut the axes at  $A, B$  and  $C$ . Prove that the locus of the foot of the perpendicular from  $O$  to the plane  $ABC$  is

$$(x^2 + y^2 + z^2)^2 (x^{-2} + y^{-2} + z^{-2}) = 4r^2 \quad 5$$

- (b) Prove that the tangents at the extremities of a diameter of an ellipse are parallel to the conjugate diameter. 5

- (c) Find the length of the semi-axes of the conic

$$ax^2 + 2hxy + ay^2 = d \quad 5$$

Or

Show that the locus of the middle points of the normal chords of the parabola,  $y^2 = 4ax$  is

$$y^4 - 2a(x - 2a)y^2 + 8a^4 = 0$$

- (d) Prove that the lines  $x = pz + q$ ,  $y = rz + s$  intersect the conic

$$z = 0, \quad ax^2 + by^2 = 1$$

$$\text{if } aq^2 + bs^2 = 1.$$

5

Or

Prove that the plane  $ax + by + cz = 0$  cuts the cone  $yz + zx + xy = 0$  in perpendicular lines, if

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

4. (a) Find the equation of the asymptotes of a central conic given by the general equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

and show that the angle between the asymptotes is

$$\tan^{-1} \left[ \frac{2\sqrt{h^2 - ab}}{a + b} \right]$$

6+4=10

Or

Obtain the equation of the chord of the conic

$$\frac{l}{r} = 1 + e \cos \theta$$

joining the two points on the conic whose vectorial angles are

$$\alpha + \beta \text{ and } \alpha - \beta$$

Hence deduce the equation of the tangent at the point, whose vectorial angle is  $\alpha$ . 8+2=10

- (b) Obtain the length and equations of the shortest distance between the lines

$$\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n}$$

$$\text{and } \frac{x - \alpha'}{l'} = \frac{y - \beta'}{m'} = \frac{z - \gamma'}{n'}$$

Find the condition for which the lines are coplanar. 6+3+1=10

- (c) Find the condition that the plane

$$lx + my + nz = p$$

may be a tangent plane to the conicoid  $ax^2 + by^2 + cz^2 = 1$  and find the coordinates of the point of contact. Also find the equation of the tangent planes to the conicoid. 6+2+2=10

( 6 )

Or

Find the area of the section of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

by the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad 10$$

(d) Show that the equation

$$(ab - h^2)(ax^2 + 2hxy + by^2 + 2gx + 2fy) + af^2 + bg^2 - 2fgh = 0$$

represents a pair of straight lines and that these straight lines form a rhombus with the line

$$ax^2 + 2hxy + by^2 = 0$$

provide that

$$(a - b)fg + h(f^2 - g^2) = 0 \quad 10$$

Or

The origin of a rectangular coordinate system is translated to the new origin  $(\alpha, \beta)$  and then the axes are rotated

( 7 )

through an angle  $\theta$ . If  $(x, y)$  and  $(x', y')$  are the coordinates of a point with respect to the original and the transformed coordinate system, respectively then show that

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

\*\*\*