

3 (Sem-1) MAT M 2

Bijni College Library
2012 P.O.-Bijni, Dist.-Chirang
(B.T.A.D) Assam

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10
- (a) Write down the n th derivative of $\log(x+1)$.
 - (b) What is the degree of the homogeneous function $f(x, y) = x^2y^3\phi(x/y) + x^5\psi(y/x)$?
 - (c) Write down the subtangent for the curve $\log y = x \log a$.
 - (d) Find the radius of curvature at any point (s, ψ) on the curve $s = c \log \sec \psi$.
 - (e) Write the equation of the asymptote to the curve $xy - ax - by = 0$ parallel to x -axis.

(f) If $f(x, y) = \log(x^2 + y^2)$, find $\frac{\partial f}{\partial x}$.

(g) If $I = \int \frac{dx}{(1-x)\sqrt{x}}$, find I .

(h) What is the value of $\int_0^{\pi} \frac{\sin 4x}{\sin x} dx$?

(i) Evaluate :

$$\int_{-\pi/2}^{\pi/2} x^9 \cos x dx$$

(j) Find the complete perimeter of the curve

$$x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$$

2. Answer the following questions : 2×5=10

(a) If $y = \cos^3 x$, find y_n .

(b) Prove that the pedal equation of the curve $r = e^\theta$ is $2p^2 = r^2$.

(c) If $I_n = \int_0^{\pi/2} \sin^n x dx$, prove that

$$I_n = \frac{n-1}{n} I_{n-2}, n > 1$$

(d) Prove that the length of the arc of the curve given by $x = e^\theta \sin \theta$, $y = e^\theta \cos \theta$ between the points $\theta = 0$ and $\theta = \frac{\pi}{2}$ is $\sqrt{2}(e^{\pi/2} - 1)$.

(e) Show that the area enclosed by the curve $r = a \cos 2\theta$ is $\frac{1}{2} \pi a^2$.

3. Answer the following questions : 5×4=20

(a) If $u = f(r)$, where $r = \sqrt{x^2 + y^2}$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

(b) Trace the curve $x^2 y^2 = x^2 - a^2$.

Or

If $p = x \cos \theta + y \sin \theta$ touches the curve

$$\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1$$

prove that $p^n = (a \cos \theta)^n + (b \sin \theta)^n$.

(c) Integrate :

$$\int \frac{dx}{3 + 2 \sin x + \cos x}$$

Or

$$\int \frac{dx}{(1+x)\sqrt{1+2x-x^2}}$$

- (d) Show that the volume of the solid produced by the revolution of the loop of the curve about the x -axis is

$$2\pi a^3 \left(\log 2 - \frac{2}{3} \right)$$

Or

Show that the volume of a right circular cone of height h and base of radius a is $\frac{1}{3}\pi a^2 h$.

4. Answer the following questions : 10×4=40

- (a) (i) If u is a homogeneous function of x and y of degree n having continuous partial derivatives, prove that

$$\left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^2 u = n(n-1)u$$

- (ii) If $u = \cos^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, prove by using

Euler's theorem on homogeneous function that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \cot u \quad 6+4=10$$

Or

Prove the following : 5+5=10

$$(i) \quad x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

where $y = a \cos(\log x) + b \sin(\log x)$.

$$(ii) \quad D^n (\tan^{-1} x) = (-1)^{n-1} \underline{n-1} \sin^n \theta \sin n\theta$$

where $\theta = \cot^{-1} x$.

- (b) A curve is represented in polar coordinates (r, θ) by the equation $r = f(\theta)$. Show that radius of curvature at the point $P(r, \theta)$ on the curve is given by

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - r_2}$$

where $r_1 = f'(\theta)$ and $r_2 = f''(\theta)$. Hence or otherwise show that if ρ_1 and ρ_2 are the radii of curvature at the extremities of any focal chord of the cardioid $r = a(1 + \cos\theta)$, which passes through

the pole, then $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$. 5+5=10

Or

Define cusp, isolated points, single cusp and double cusp. Find the position and nature of the multiple points on the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$.

4+6=10

- (c) (i) If $I_{m, n} = \int \cos^m x \cos nx dx$, prove that

$$I_{m, n} = \frac{\cos^m x \sin nx}{m+n} + \frac{m}{m+n} I_{m-1, n-1}$$

- (ii) If $U_n = \int_0^{\pi/2} \theta \sin^n \theta d\theta$ and $n > 1$, then prove that

$$U_n = \frac{n-1}{n} U_{n-2} + \frac{1}{n^2} \quad 5+5=10$$

- (d) (i) Find the area included between the curve $y^2(a-x) = x^3$ and its asymptote.

- (ii) Show that the surface area of the solid generated due to the revolution of the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$ about the x -axis is $\frac{6}{5} \pi a^2$. 5+5=10

3 (Sem-1) ENG (Sc)

2012

Bijni College Library,
P.O.-Bijni, Dist.-Chirang
(B.T.A.D) Assam

ENGLISH
(Science)

(Functional English—I)

Full Marks : 40

Time : 2 hours

*The figures in the margin indicate full marks
for the questions*

1. (a) Write the meanings of the underlined abbreviations :

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(i) I heard my name called out on the PA system.

(ii) May I introduce Sheila, who is Ms. Brown's PA.

(iii) If you are filling in a form, put N/A if the question doesn't apply to you.

(iv) Encl. CV and photo.

(v) Ask them to reply asap.