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ECONOMICS

(Major)

Paper : 4.1

(**Mathematical Application in Economics**)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions (as directed) :
1×10=10

(a) Given the following total cost function

$$TC = 1000 + 2x - 5x^2$$

Find the average fixed cost.

(b) Obtain the marginal revenue function
from the following total revenue
function :

$$R = 100 - \frac{1}{\sqrt{Q}} + Q^2$$

- (c) Given the marginal cost function

$$MC = 4 - 0.4x$$

and total fixed cost being 100, find total cost when output $x = 10$.

- (d) A firm has the following price and total cost function

$$TR = 100q - 4q^2$$

$$TC = 50 + 20q$$

find the profit function if government imposes a specific sales tax at the rate of 8.

- (e) What does the word 'person' imply in a two-person zero-sum game?
- (f) A game with no saddle point is solved by adopting — (mixed/pure/optimal) strategies.

(Choose the correct one)

- (g) The demand curve $P = a$, under perfect competition has infinite elasticity.

(Write True or False)

- (h) The rate of capital formation over time can be considered as net investment flow over time.

(Write True or False)

- (i) Linear programming problems are always stated in linear relations.

(Write True or False)

- (j) Give an example of economic problem, that can be solved by the technique of linear programming.

2. Answer the following questions : 2×5=10

- (a) Given the market model

$$D = 30 - P^2$$

$$S = -15 + 4P^2$$

$$D = S$$

where D , S , P denote demand, supply and price respectively. Obtain the equilibrium price and quantity.

- (b) The marginal cost of a firm is given by

$$MC = 3q^2 - 10q + 6$$

with total fixed cost equal to 100, find the average cost when output is 5.

- (c) If the marginal revenue of a firm is ₹ 25 and price elasticity of demand is 2, find its average revenue.

- (d) What is a pay-off matrix?

- (e) Given the demand function

$$D = \frac{C}{P}$$

where D is demand, P is price and C is constant. Obtain the price elasticity of demand.

3. Answer any four of the following : 5×4=20

(a) If $xy = a$, show that

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 0$$

(b) The total cost function of a firm is given by

$$TC = 300x - 10x^2 + \frac{1}{3}x^3$$

find the optimum level of output of the firm.

(c) State Euler's theorem. Show that the Cobb-Douglas production function

$$Q = AK^\alpha L^\beta$$

satisfies the Euler's theorem, if $\alpha + \beta = 1$.

(d) Using differential calculus, establish the following relationship among marginal revenue (MR), average revenue (AR) and elasticity of demand (e_d)

$$e_d = \frac{AR}{AR - MR}$$

(e) Under pure competition, find the consumer's surplus defined by the demand curve $D(X) = 16 - x^2$ and supply curve $S(X) = 4 + x$.

(5)

(f) If the marginal cost function is

$$C'(Q) = 2 + 3\sqrt{Q} + \frac{5}{\sqrt{Q}}$$

find the total cost function $C(Q)$, when $C(1) = 21$.

4. Answer the following questions : 10×4=40

(a) The production function of a commodity is given by

$$Q = -3L^3 + 18L^2$$

where Q is output and L depicts the units of inputs.

(i) Find the number of units of input required to give maximum output.

(ii) Verify that when the average product (AP) is maximum, it is equal to the marginal product (MP).

$$5+5=10$$

Or

Prove that neither a lump sum, nor a profit tax will affect the optimum price-quantity combination for a monopolist. 10

- (b) The total revenue and total cost functions of a monopolist are

$$TR = 46Q - 3Q^2$$

$$TC = 2Q^2 - 4Q + 10$$

where Q is output. An excise tax at the rate of t is imposed on output. Find—

- (i) the tax rate t which will maximize total excise revenue of the government;
- (ii) the monopolist's maximum profit after the payment of tax and profit maximizing price and output. $5+5=10$

Or

A monopolist has the following demand and total cost functions :

$$P = 100 - Q$$

$$TC = 50 - 2Q + 4Q^2$$

where P is price and Q is quantity of output. If the government imposes a specific sales tax at the rate of ₹ 2 per unit of output, find—

- (i) profit maximizing output after payment of tax;
- (ii) show the effect of tax on equilibrium output, price and profit. $5+5=10$

- (c) (i) For what value of λ , the game with the following pay-off matrix is strictly determinable?

Player A	Player B		
	B_1	B_2	B_3
A_1	λ	6	2
A_2	-1	λ	-7
A_3	-2	4	λ

- (ii) Find the optimal strategies and value of the game whose pay-off matrix is

Player A	Player B			
	B_1	B_2	B_3	B_4
A_1	1	7	3	4
A_2	5	6	4	5
A_3	7	2	0	3

5+5=10

Or

Solve the following linear programming problem by graphic method :

$$\text{Maximize } R = x + 4y$$

subject to

$$2x + 3y \leq 4$$

$$3x + y \leq 3$$

$$\text{and } x, y \geq 0$$

10

- (d) The price and total cost functions of a price discriminating monopoly are given by

$$P_1 = 80 - 5Q_1$$

$$P_2 = 180 - 20Q_2$$

$$TC = 50 + 2Q \quad (Q = Q_1 + Q_2)$$

Q_1 and Q_2 are the outputs to be sold in the two markets. Find—

- (i) profit maximizing output;
 (ii) equilibrium prices of the two markets;
 (iii) maximum profit;
 (iv) elasticity of demand of the two markets. 5+2+1+2=10

Or

A monopolist produces his output in two plants and his total output is $Q = Q_1 + Q_2$. His cost functions are given by

$$C_1 = 10Q_1$$

$$C_2 = 0.25Q_2^2$$

The demand curve of the monopolist is given by

$$Q = 200 - 2P$$

Find profit maximizing outputs and maximum profit of the monopolist. 10
