2013

ECONOMICS

(Major)

Paper : 3.1

(Elementary Mathematics for Economics)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) If $U = \{5, 6, 7, 8, 9\}$ and $A = \{7, 8\}$, find the complement of A, i.e., $\tilde{A}$.

(b) The set of all real numbers is greater than 8 but less than 73. Write in set notation.

(c) Differentiate between 'domain' and 'range' of a function.

(d) Give an example of rectangular hyperbola.

(e) Find the limit of the function $\lim_{x \to 1} f(x) = \lim \frac{1-x}{1-x^2}$.

(f) Define 'dimension' or 'order' of a matrix.

(g) Find the transpose of A, given $A = \begin{bmatrix} 1 & 0 & 9 \\ 6 & 1 & 2 \end{bmatrix}$

(h) State the power rule of integration.

(i) Define rank of a matrix with example.

(j) Distinguish between a singular matrix and a nonsingular matrix.

2. Answer the following questions : $2 \times 5 = 10$

(a) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$, compute $2A - 3B$.

(b) State the conditions for equality of two matrices.

(c) If $C = 1000 + 6x + 0 \cdot 5x^2$, where C is total cost and x is output, find marginal cost (MC).

(d) Given $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 5 & 1 \end{bmatrix}$, show that $(A + B)' = A' + B'$.

(e) Solve :

$$\int (2x^2 + x^2)\,dx$$

3. Answer briefly any *four* of the following : 5×4=20

(a) Prove that for any two scalars g and k—

(i) $k(A + B) = kA + kB$;

(ii) $(g + k)A = gA + kA$. 2½+2½=5

(b) State the power rule of differentiation. Given $y = \sqrt{x}$, find $\frac{dy}{dx}$. 2+3=5

(c) Find the derivative of the function

$$Y = f(x) = (10 + 5x^2)2x^3$$

using product rule.

(d) Find the cofactors of the matrix

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 5 & 2 \end{bmatrix}$$

(e) Given $y = f(x_1, x_2) = (x_1^2 + 5)(2x_1 - x_2^2)$, find $\frac{\delta y}{\delta x_1}$ and $\frac{\delta y}{\delta x_2}$.

(f) Find AB, given

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 2 & 3 \\ 4 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 1 & 2 \\ 2 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}$$

4. Answer any *four* of the following : $10 \times 4 = 40$

(a) (i) A function is defined as follows :

$$\begin{aligned} f(x) &= 1, \quad x > 1 \\ &= 0, \quad x = 0 \\ &= -1, \quad x < 0 \end{aligned}$$

Show that the function is discontinuous at $x = 0$. 5

(ii) State the conditions of continuity of a function. Differentiate between $f(a)$ and $\lim_{x \to a} f(x)$. $3+2=5$

(b) Solve the following equation system by using Cramer's rule :

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 15 \\ x_1 - 2x_2 + 5x_3 &= 13 \\ 4x_1 + 3x_2 - x_3 &= 11 \end{aligned}$$

10

(c) State five properties of determinants. $2 \times 5 = 10$

(d) Solve the following market model using matrix inversion : 10

$$Q_d = 10 - 0 \cdot 4P$$
$$Q_s = -3 + 0 \cdot 6P$$
$$Q_d = Q_s$$

(e) (i) State and prove the quotient rule of differentiation. 5

(ii) If $y = \frac{2x-3}{x+1}$, find $\frac{dy}{dx}$ using quotient rule. 5

(f) (i) Find the definite integral :

$$\int_1^3 (4x - x^2 - 3)\, dx$$

5

(ii) Given the marginal revenue (MR) function, i.e., $R'(Q) = 50 - 4Q$, find the total revenue (TR) function, i.e., $R(Q)$. 5

(g) Discuss the structure of a static open input-output model. State its assumptions. 7+3=10

(h) Solve the input-output model $(I - A)X = F$ by using either matrix inversion or Cramer's rule, given

$$A = \begin{bmatrix} 0{\cdot}2 & 0{\cdot}2 & 0 \\ 0{\cdot}3 & 0{\cdot}2 & 0{\cdot}4 \\ 0{\cdot}2 & 0{\cdot}3 & 0{\cdot}1 \end{bmatrix}; \quad F = \begin{bmatrix} 100 \\ 220 \\ 150 \end{bmatrix}$$ 10

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